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COLOUR VISION AS A FUNCTION OF A COMPLEX VARIABLE*

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An algebra of colour vision somewhat resembling that of spinors is proposed. In this algebra white colour corresponds to zero and the colour pattern to complex numbers. Thus, the intensity of colour is the amplitude of a complex number and colour is a phase. The Young–Helmholtz and Hering schemes are considered. It is shown that both these schemes ultimately lead to the same algebra of complex numbers, although the projective-geometric algorithms generating them differ. Copyright © 1996 Elsevier Science Ltd.

Complementary summation of colours is one of the most impressive phenomena of human vision. The sum of colours situated at opposite ends of any diameter of a coloured circle gives the colour white [1]. The sum of any three colours situated at an angle 120° to each other in this circle also gives the colour white. The "quantity" of mutually compensated colours giving a white colour changes the saturation of the colour but not the colour itself and not its intensity [2, 3]. According to the Young–Helmholtz theory, three main colours exist (primary red, primary green and primary azure) present on a single circle at an angle of 120° to each other. According to the Hering theory considered for a long time as an alternative, all colours are generated by four colours, namely blue, yellow, ortho-red and ortho-green, which on a colour circle form two mutually orthogonal axes. Much experimental material has now been gathered complicating this picture. Thus, in the experiments of Land and also in experiments with so-called Mondrian-like pictures it was shown that the light perceived does not depend on the spectrum of the source [3, 4]. Thus, in the course of processing the signals arriving at the retina, their nontrivial transformations take place [5] one of which is the formation of the vector field on a plane of three or four positive signals. The present paper is devoted to analysis of these transformations.

The mathematical mean of white colour is obvious: in two-dimensional vector space of colours (in which the colours on the colour circle summate by vectors) it is zero. If as a result of transformations of the signals forming the colour circle they not only summate but multiply, then each point on the colour plane represents a complex number. The intensity of the colour is the amplitude of this complex number, while colour is its phase. The Young–Helmholtz and Hering schemes become equivalent: they correspond to two modes of choice of the basis. This basis, however, is somewhat unusual. Instead of the real axes of traditional Cartesian coordinates, the

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basis is formed by rays, the points on which take values from zero to infinity. The reason for this is that, for example, no negative yellowishness exists — it corresponds to a positive blue colour. The same applies to any colour (for each colour a complementary colour exists, i.e. giving in sum with it a white colour or in the mathematics considered zero) but this is a different colour and only the rule of annihilation determines what it is. This algebra is unlike school algebra where minus five apples is certainly equal to apples. As will be shown below the algebra is generated naturally by projective geometry.

COMPLEMENTARY NUMBERS

Let us begin with a two-dimensional projective transformation. Let us consider a set of pairs of numbers $a, b \geqslant 0$. *In toto* they form a set of vectors $\binom{a}{b}$ of the first quadrant of the plane (a, b). The sum of any two vectors of the first quadrant and also multiplication of the vector by the number $r > 0$ are obviously equal to the vector of the first quadrant. Let us define the commutative cyclic product of the vectors

$$\boldsymbol{X} = \begin{pmatrix} a \\ b \end{pmatrix} \odot \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac + bd \\ ad + bc \end{pmatrix} = (ac + bd)\boldsymbol{r}_1 + (ad + bc)\boldsymbol{r}_{-1}, \tag{1}$$

which brings into correspondence to the two arbitrary vectors of the first quadrant a vector also belonging to the first quadrant. Then let us bring into correspondence to each vector $\boldsymbol{X}$ another vector $\boldsymbol{Y} = \veebar \boldsymbol{X}$ belonging either to the semi-axis $\binom{a}{0}$ or the semi-axis $\binom{0}{b}$ by the following rule:

$$\veebar \begin{pmatrix} a \\ b \end{pmatrix} = \begin{cases} \begin{pmatrix} a - b \\ 0 \end{pmatrix}, & \text{if} \quad a \geqslant b \\ \begin{pmatrix} 0 \\ b - a \end{pmatrix}, & \text{if} \quad b > a. \end{cases} \tag{2}$$

Let us call the vector $\boldsymbol{Y}$ the complementary projection $\veebar$ of the vector $\boldsymbol{X}$ or, in short, a complementor. It may be shown that the complementor of the sum of the vectors (complementary sum $\oplus$) is equal to the sum of the complementors of these vectors and the complementor of the cyclic product of two vectors $\veebar(\boldsymbol{X}_1 \odot \boldsymbol{X}_2) \equiv \boldsymbol{X}_1 \otimes \boldsymbol{X}_2$ (the complementary product $\otimes$) is equal to the cyclic product of the complementors of these vectors. Each complementor as well as zero has a single complementor reciprocal to it by multiplication

$$\begin{pmatrix} a \\ b \end{pmatrix}^{-1} = \begin{cases} \begin{pmatrix} 1/(a - b) \\ 0 \end{pmatrix}, & \text{if} \quad a \geqslant b \\ \begin{pmatrix} 1 \\ 1/(b - a) \end{pmatrix}, & \text{if} \quad b > a. \end{cases} \tag{3}$$

The complementary quotient of two complementors $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ is defined as the complementor of the product: $\boldsymbol{X}_1 \oslash \boldsymbol{X}_2 \equiv \boldsymbol{X}_1 \otimes \boldsymbol{X}_2^{-1}$ and the complementary difference from the formula

$$\boldsymbol{X}_1 \ominus \boldsymbol{X}_2 = \boldsymbol{X}_1 \oplus \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \boldsymbol{X}_2\right]. \tag{4}$$

Thus, in this algebra the unit is the vector $\boldsymbol{r} = \binom{1}{0}$, the minus unit is the vector $\boldsymbol{r}_{-1} = \binom{0}{1}$ and the zero is the vector $\binom{0}{0}$. It may be shown that the complementary (projective) operations with two-dimensional vectors with non-negative components (patterns) in general and with complementors in particular are isomorphous to the algebra of real numbers. In this representation a real number is the projection of a point parallel to the bisector of the first quadrant of the plane (Fig. 1) and multiplication of the vector by -1 corresponds to transposition of the semi-axes x and y. The complementary sum and the complementary product of two vectors belonging to two different semi-axes is a vector which may belong to any of the two semi-axes. These projective transformations with two-dimensional matrices *in toto* form a complementary arithmetic, the rules of which concur with the usual arithmetic.

Of course, it is possible to determine the functions of the complementary two-dimensional vectors. Transformation of such functions are equivalent to transformations of the functions of a real variable. It is also possible to determine the transformation of the first quadrant, leaving invariant the complementary projections. It has the form

$$a' = \frac{a - b\,\mathrm{tg}\Psi}{1 + \mathrm{tg}\Psi},\ b' = \frac{b - a\,\mathrm{tg}\Psi}{1 + \mathrm{tg}\Psi}. \tag{5}$$

With this transformation the axes turn in the direction of the bisector through identical angles. We do not overlook the fact that the transformation of the angles of turn for two successive complementary transformations coincides with the corresponding Lorentz transformation.

FOUR-DIMENSIONAL COMPLEMENTARY VECTORS AND THE HERING SCHEME

Let us determine a 4-vector, complementary to the given 4-vector (four-dimensional complementor) as follows:

$$\underline{\vee}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \underline{\vee}\ (a\mathbf{c}_1 + b\mathbf{c}_i + c\mathbf{c}_{-1} + d\mathbf{c}_{-i}) = \tag{6}$$

$$\begin{cases} (a-c)\mathbf{c}_1 + (b-d)\mathbf{c}_i, & \text{if} \quad a \geqslant c \quad \text{and} \quad b \geqslant d \\ (c-a)\mathbf{c}_{-1} + (b-d)\mathbf{c}_i, & \text{if} \quad c > a \quad \text{and} \quad b \geqslant d \\ (c-a)\mathbf{c}_{-1} + (d-b)\mathbf{c}_{-i}, & \text{if} \quad a \geqslant c \quad \text{and} \quad d > b \\ (c-a)\mathbf{c}_{-1} + (d-b)\mathbf{c}_{-i}, & \text{if} \quad c > a \quad \text{and} \quad d > b. \end{cases}$$

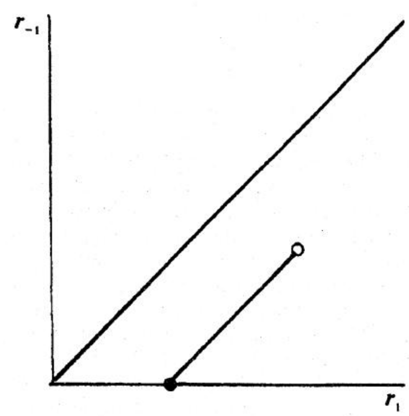

Fig. 1. Complementary projection on a plane parallel to a white ray (bisector of the first quadrant).

Thus, in a four-dimensional complementor not more than two components differ from zero. It is determined by projection parallel to the bisectors of the first quadrant in the planes (c_1, c_{-1}) and (c_i, c_{-i}) (Fig. 2). Let us determine the cyclic product of the vectors:

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \odot \begin{pmatrix} e \\ f \\ g \\ h \end{pmatrix} \equiv \begin{pmatrix} ae + bh + cg + df \\ be + ch + dg + af \\ ce + dh + ag + bf \\ de + ah + bg + cf \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = x\boldsymbol{c}_1 + y\boldsymbol{c}_i + z\boldsymbol{c}_{-1} + w\boldsymbol{c}_{-i}, \tag{7}$$

and also the complementary product

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \otimes \begin{pmatrix} e \\ f \\ g \\ h \end{pmatrix} \equiv \underline{\vee} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \odot \begin{pmatrix} e \\ f \\ g \\ h \end{pmatrix} = \begin{cases} (x-z)\boldsymbol{c}_1 + (y-w)\boldsymbol{c}_i, & \text{if } x \geqslant z \text{ and } y \geqslant w \\ (x-z)\boldsymbol{c}_1 + (w-y)\boldsymbol{c}_{-i}, & \text{if } x \geqslant z \text{ and } w > y \\ (z-x)\boldsymbol{c}_{-1} + (y-w)\boldsymbol{c}_i, & \text{if } z > x \text{ and } y \geqslant w \\ (z-x)\boldsymbol{c}_1 + (w-y)\boldsymbol{c}_{-i}, & \text{if } z > x \text{ and } w > y \end{cases} \tag{8}$$

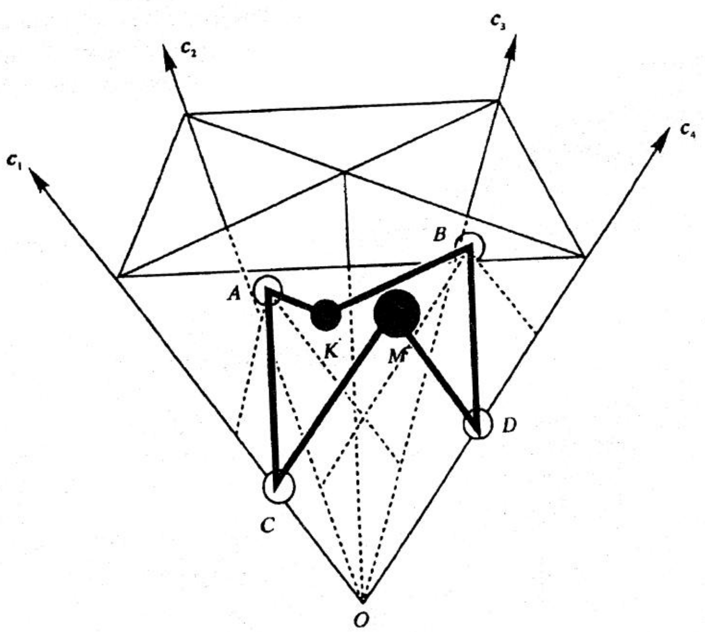

Fig. 2. Formation of a colour circle by the Hering scheme occurs as follows. The four base rays determine the four-vector (a,b,c,d). In all, there are six combinations of pairs of four base rays, forming base quadrants in four-dimensional space. One of them ("real") is separated out, which immediately determines the quadrant (imaginary) complementary to it. The remaining four quadrants ("complex") form a cycle. This cyclic structure of space finds its reflection in the structural product. The complementary algebraic operations correspond to the passage of the six-member, pathway into the spaces of the images. 1. The four-dimensional pattern K is projected on to the real and imaginary base quadrants formed by pairs of base rays, respectively. Thus, the projections A and B are obtained. 2. In the "real" and "imaginary" planes, complementary projection takes place. Thus, two complementors C and D are obtained. 3. From a pair of complementors is constructed the pattern M in four-dimensional space (in reality belonging to one of the four base quadrants), which is also a complementary number. The complementor C determines the real part of the complex number and the complementor D determines its imaginary part.

of 4D vectors which by definition are a pattern of the complementary projection $\veebar$ of the cyclic product. This is a 4D vector belonging to one of the four positive quadrants (c_1, c_i), (c_i, c_{-i}), (c_{-1}, c_{-i}) and (c_{-i}, c_1) formed by the basis rays on to which there is no projection. The complementors are defined not on the plane as the usual complex numbers but on four quadrants orthogonal to each other in four-dimensional space. To each point in the space of the patterns corresponds a complementor on one of the basis quadrants, although the complementary arithmetical operations transform the complementors as though they were complex numbers.

As was done above for two-dimensional vectors, complementary summation, subtraction and division are determined. Direct calculation shows that $c_i \otimes c_i = c_{-1}$ and $c_{-1} \otimes c_{-1} = c_1$. Thus, c_{-1} in this algebra is the minus unit and c_i the root of the minus unit. It may be demonstrated that the arithmetical operations with four-dimensional complementary vectors defined in this paragraph are isomorphous to the algebra of complex numbers [6, 7]. If multiplication of complementary vectors does not take place, then four-dimensional complementary arithmetic generates vectors on the plane corresponding to the Hering scheme. The four semi-axes of the four-dimensional space with non-negative coefficients are the four basis colours (blue, yellow, ortho-red and ortho-green). The intensity is the amplitude of a complementary four-dimensional complex number and colour its phase. In respect of the correspondence of colours to one or other basis vectors some arbitrariness exists. As long as this correspondence has not been established experimentally, we agree to consider that the minus unit in this algebra corresponds to the colour yellow and the imaginary unit to the ortho-red colour.

THREE-DIMENSIONAL COMPLEMENTORS AND THE YOUNG–HELMHOLTZ SCHEME

Let a vector in three-dimensional positive space, i.e. in the space of 3D vectors with non-negative components be given. Let us determine the three-dimensional complementor:

$$\veebar \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \veebar (ae_1 + be_{2\pi i/3} + ce_{-2\pi i/3}) = \begin{cases} \begin{pmatrix} a-c \\ b-c \\ 0 \end{pmatrix}, & \text{if} \quad a \geqslant c \text{ and } b \geqslant c \\ \begin{pmatrix} a-b \\ 0 \\ c-b \end{pmatrix}, & \text{if} \quad a \geqslant b \text{ and } c \geqslant b \\ \begin{pmatrix} 0 \\ b-a \\ c-a \end{pmatrix}, & \text{if} \quad b \geqslant a \text{ and } c \geqslant a. \end{cases} \tag{9}$$

and the cyclic product

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \odot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \begin{pmatrix} ad + bf + ce \\ bd + cf + ae \\ cd + af + be \end{pmatrix}. \tag{10}$$

As was done above for two- and four-dimensional vectors, one may define the complementary arithmetical operations:

$$\begin{aligned} &X_1 \otimes X_2 = \veebar (X_1 \odot X_2),\ X_1 \oplus X_2 = \veebar (X_1 + X_2), \\ &X_1 \ominus X_2 = \veebar (X_1 + e_{-1} \otimes X_2),\ X_1 \oslash X_2 = X_1 \otimes X_2^{-1} \end{aligned} \tag{11}$$

Here, as above, the vector X^{-1} complementarily reciprocal to the given one by definition is such that $(X^{-1} \otimes X) = (X \otimes X^{-1}) \equiv \veebar (XX^{-1}) = e_1$. It may be demonstrated that operations with three-dimensional complementary vectors are isomorphous to the algebra of complex numbers. The minus unit in this algebra is the vector $e_{-1} = e_{-\frac{2\pi i}{3}} + e_{\frac{2\pi i}{3}}$ and the imaginary unit is $e_i = \frac{1}{\sqrt{3}}(e_1 + 2e_{\frac{2\pi}{3}i})$. The base vectors $e_{\frac{2\pi}{3}i}$ and $e_{-\frac{2\pi}{3}i}$ are linked by the relations

$$e_{2\pi/3} = e_{-2\pi/3} \otimes e_{-2\pi/3},\ e_{-2\pi/3} = e_{2\pi/3} \otimes e_{2\pi/3},$$
$$e_{2\pi/3} \otimes e_{-2\pi/3} = e_{-1} \otimes e_{-1} = e_1,\ e_i \otimes e_i = e_{-1}. \tag{12}$$

It should be noted that unlike the traditional definition of a complex number as a pair of real numbers, the triad scheme defines complex numbers directly (Fig. 3). The negative real number in this scheme is equal to the sum of complex numbers $e_{\frac{2\pi}{3}i}$ and $e_{-\frac{2\pi}{3}i}$ which also differs from the traditional approach.

The arithmetical operations with complementors in the base quadrants orthogonal to each other, though equivalent to the operations with complex numbers, nevertheless appear non-

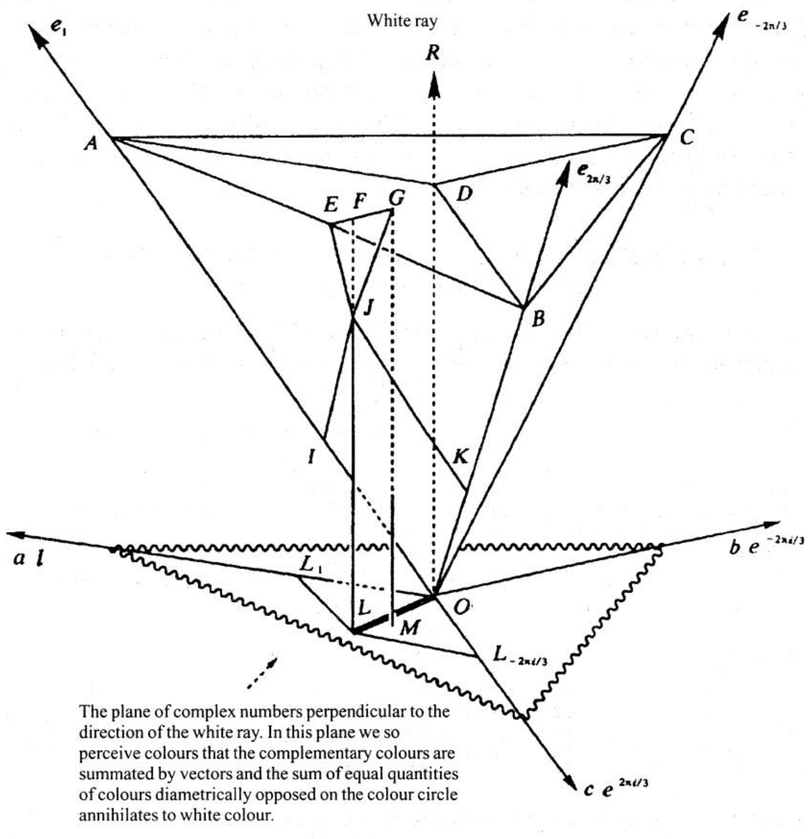

Fig. 3. Geometrically, the correspondence of the three-dimensional pattern to complex numbers by the triad scheme occurs as follows: 1. The pattern F (vector in the first quadrant of three-dimensional space) is brought into correspondence to each triplet of non-negative numbers. 2. There is complementary projection parallel to the "white ray" (i.e. to the ray, forming with each of the three semi-axes, equal angles) on to one of the three complementary quadrants. Thus, the complementor J is brought into correspondence to each point of the first octant F. 3. There is complementary projection L of the point F on to the plane perpendicular to the white ray and passing through the origin of the coordinates.

customary. Thus, Fig. 4 presents the sector of the single complementary circumference in the base quadrant obviously not a circumference in the customary sense. In the complementary representation, the complex plane is formed by three quadrants, the sum of the angles of which is equal to only 270°. Only the second projection — on the plane perpendicular to the white ray (which is projected to zero on this plane) — imparts the customary form to the single circle, the complementary numbers and the complex plane as a whole. In this "projection" of the three-dimensional pattern the colours may summate by vectors and the colour circle looks like a circle. On formation of the colour image the ray equidistant from the base rays of the positive octant corresponds to the colour white. The minus unit and the imaginary unit are obtained as a result of transformation corresponding to passage from Young–Helmholtz's base rays to Hering's four base rays.

DISCUSSION OF THE RESULTS

"Can persons see anything. . . apart from shadows, cast by a fire on to the cave wall situated in front of them?" (Plato, Gosudarstvo Publ.).

The projective-geometric approach with complementary algebraic operations of transformation of signals seems to us quite natural for explaining the phenomenon of colour vision. We do not see any alternative explanation: formation of the whole set of colours from three primary; the formation of a colour circle with vector summation of colours; "annihilation" of complementary colours into a white and only white colour; the resolution of white and only white colour into the colours of the spectrum; transformation of all the colours of the rainbow arranged by frequency linearly into three-quarters of the colour circle; correspondence of part of the colour circle not to any pure colours of the spectrum but combinations of pairs of primary colours with different intensities; dependence of the saturation of colour on the amount of "whiteness" in it; equivalence

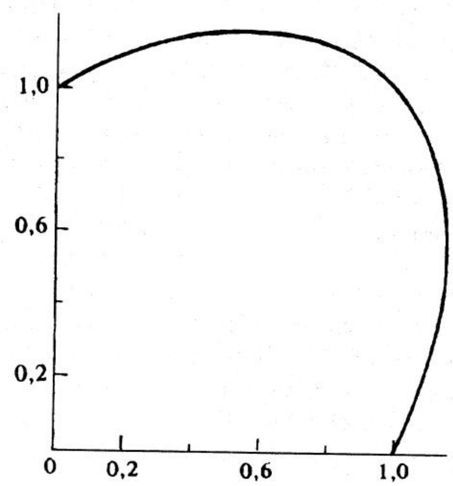

Fig. 4. Part of the single circle corresponding to the angle $2\pi/3$ of the complex plane in projection on to the base quadrant. In this projection the "single circle", in the usual sense is not part of the circumference. The functions on the complex plane assume the usual form after the second projection, as shown in Fig. 3.

at physiological level of the Young–Helmholtz three-colour scheme and Hering's four-colour scheme.*

The fact that, on processing the visual pattern, there is projective transformation in itself is not surprising. The visual pattern is the projection of the three-dimensional world on to the two-dimensional surface of the eye fundus. The volumetric image, which man sees with two eyes is a process opposite to projection, namely the construction of a three-dimensional pattern from a two-dimensional image. Such calculations are not at all necessarily made by the same scheme as in computers. Thus, the positional system of calculation may be used or not used (and most probably is not used) for any calculations *in vivo*.

Next, it is certainly not obvious that Boolean algebra, on which are based calculations in modern computers, is used *in vivo* for the calculations. In the context of the results of this paper, the question of what is absolutely necessary for the calculations, be it *in vivo* or in design, appears to be quite pertinent. A detailed discussion of the general principles of the calculations goes beyond the scope of the present paper. But with generality, which for our purposes appears sufficient, it is natural to assume that some type of algebra must be present in all computations, be it in a computer, in an extraterrestrial civilization a milliard light years from the Earth or in the brain of the reader of this paper. In other words, for any computations, including the processes of recognition or pattern construction, the operations of multiplication and summation in one way or another must be determined in the same way as finding an element as a result of back transformation.

The number of primary colours, as is known, is equal to three. However, three-dimensional algebra does not exist. The sole two-dimensional algebra is the algebra of complex numbers. And this is precisely that algebra of colours to which lead in natural fashion the transformations leading to the formation of the colour circle.

The algebra considered in the present paper is unusual because it deals with a cone of signals and not with the space of the signals (it is known that a cone in the broad sense of the word forms as a result of multiplication of the base vectors by non-negative numbers. The term "cone" is standard, although in this case the "form" of the space rather resembles an n-dimensional pyramid). Usually, mathematicians make the assumption on the possibility of multiplication by a real number at the very start of construction of the model. Apparently, this was the reason why the idea on multidimensional representations of numbers is not just as standard as, say, the representation of a complex number in the form of a pair of real numbers. In the multidimensional representation of numbers, the positive numbers are generated by a certain matrix and the negative by another matrix [7]. Historically, the properties of positivity mathematics have been far from studied in such detail as the properties of real, complex or whole numbers. On the other hand, analysis of sets of positive numbers and operations with them, at first sight, looks like a special case of operations with real numbers. These ideas allow one, to a certain degree, to resolve the quandary why this system was not developed earlier or, at least, is not widely known.

The algebra generated by a cone of positive numbers, generally speaking, is obtained by the following procedure. Firstly, a group-generator is chosen. Secondly, a regular representation of

* The projective approach developed in the present paper also allows one to consider not only point but also statistical groups in which for group transformations the objects coincide, not exactly but approximately. This is possible thanks to the fact that the proposed formalism contains not only a group but also the algebra generated by this group. Such groups may prove to be useful in many fields of biology and technology. However, the development of the formalism of "approximate symmetry" falls outside the bounds of this work and a separate paper will be devoted to it.

this group is constructed. Thirdly, linear combinations of the matrices of the regular representation are constructed with non-negative coefficients (the matrices thus obtained were termed complementary [6]). Finally, one defines a projective rule, which brings into correspondence to each point of the cone, the point of a subcone (the one-dimensional cone (subcone) is a ray, the two-dimensional subcone is the quadrant of the plane and so on).

The key idea in this approach is that the matrix $w = \sum w_i$, equal to the sum of all the matrices w_i of the regular representation of any finite group consisting of n elements, has n^2 elements, each of which is equal to unity: $w_{ij} = 1$. To construct a complementary algebra, this matrix w must, by definition, be equal to zero! In other words, the matrix w is the element of a certain subspace proper. After projective transformation, all the elements of this subspace are projected to the origin of coordinates, i.e. to zero.

One may construct a complementary algebra over the cone with matrices in the form of axes. Such an algebra, to a certain degree, resembles the algebra of spinors. However, it should not be forgotten that we have not the space of matrices but the cone of matrices.

Apparently, each complementary algebra may be generated by a cone with rays in the form of vectors, as is done in the present work, or by a cone with rays in the form of matrices. There is a mutually unambiguous correspondence between the matrix and vector representations of complementary algebra, since identical groups and identical projective transformations lead to the same algebra.

If the generator is an Abelian group consisting of two elements, projection occurs along the bisector of the cone which, in the given case, is simply the first quadrant of the plane. Such geometry leads to the algebra of real numbers. If the generator of the group is a third-order cyclic group, the projection must be parallel to the "height" of the cone. As a result we obtain the algebra of complex numbers. It should be noted that in this representation the complex numbers are obtained directly but the real ones as linear combinations of complex numbers.

If the generator is a fourth-order Abelian cyclic group, the corresponding projective procedure is defined from formula (8). Each point of the four-dimensional cone is projected on to one of four mutually perpendicular quadrants. The algebraic operations with the points of these quadrants are isomorphous to the operations with complex numbers.

If we consider an Abelian group with four elements g_1, g_2, g_3 and e, such that $g_1^2 = g_2^2 = g_3^2 = e$; $g_1g_2 = g_3$; $g_1g_3 = g_2$; $g_2g_3 = g_1$, the analogous projective procedure generates dual numbers (it is known that not each dual number has a reciprocal element).

The cone generated by a group of quaternions generates the algebra of quaternions. The general theory of rings, fields, algebras and other structures, generated in similar fashion, is now being developed.

The projective-geometric theory of colour vision may potentially have applications in different realms, from the formation and transmission of a colour image in television to artificial sensory systems, which are now being intensively explored. On the other hand, the family of known neuronal mediators is rapidly growing and the assumption that by interacting with the neurones they form a certain algebra set by the cone (i.e. their concentrations, which naturally are non-negative) is not only natural but apparently also necessary. Unlike other processes forming an image and hidden from perception, colour vision is a process of the formation of an image, for which one may construct a complementary algebra. Of course, such an algebra requires further study. But the hypothesis on the wide application of projective geometry for the calculations made by the brain with reference to the results of the present work appears natural and promising.

In conclusion, we would note that the problem posed by Plato appears quite relevant in this paper, especially if the word "shadow", no longer used in this context in the scientific literature, is replaced by the word "projection". If the projective-geometric transformations of the visual signals actually lead to colours, as to complex numbers, which it is known are a unique algebra on the plane, then that which we see is apparently the best approximation to the adequate vision of the world from that window in it, which was broken through for us by nature.

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