FOUR-DIMENSIONAL PYRAMIDAL STRUCTURE OF THE PERIODIC PROPERTIES OF ATOMS AND CHEMICAL ELEMENTS

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The recent results show that the dependence of the elements' properties on the nuclei charge has a four-dimensional structure. The geometrical interpretation of this structure results in a four-dimensional pyramidal representation of the periodic properties of elements, which have traditionally been presented on a plane since Mendeleyev's times.

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1. INTRODUCTION. The question whether there is a third dimension in the periodic properties of the elements has been under discussion since 1939. Pauling was first to introduce electronegativity index into the periodic system [1], while Allen suggested the configuration energy [2]. However as Maddox pointed out [3], neither Pauling nor Allen ever discussed an idea on the existence of three-dimensional invariants and symmetries in the table of elements.

The three-dimensional structure that displayed the periodicity of the elemental properties and had the shape of a step pyramid was introduced only in 1992 [4]. However, it has been recently proven [5] that the structure of the set of all elements has not only the third, but also even the fourth dimension. Below, the four-dimensional properties of the Periodic table are discussed.

2. THE STRUCTURE OF ELECTRONIC SHELLS FILLING AS A FUNCTION OF THE NUCLEI CHARGE. The correspondence of the four-dimensional pyramid of the elemental properties originates from the geometrical interpretation of Hund's empirical rule [6]: the term with the highest value of the spin S (within the given electronic configuration) and with the highest possible value of the orbital moment L (with the given S) has the lowest energy [7]. The order of the increase in energy of atomic orbitals has been experimentally proven and well established [8]:

$$\mathbf{1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f < 5d < 6p < 7s < 5f < 6d < 7p < 8s} \quad (1)$$

If one splits the set of inequalities (1) and substitutes inequality symbols by arrows, the order of orbitals filling as the function of the nuclei charge can be presented in the following form:

$$\begin{gathered}\mathbf{1s \rightarrow} \\ \mathbf{2s\rightarrow} \\ \mathbf{2p\rightarrow 3s\rightarrow} \\ \mathbf{3p\rightarrow 4s\rightarrow} \\ \mathbf{3d\rightarrow 4p\rightarrow 5s\rightarrow} \\ \mathbf{4d\rightarrow 5p\rightarrow 6s\rightarrow} \\ \mathbf{4f \rightarrow 5d \rightarrow 6p \rightarrow 7s \rightarrow} \\ \mathbf{5f \rightarrow 6d \rightarrow 7p \rightarrow 8s}\end{gathered} \quad (2)$$

If one regroups the members again and changes the line order for the opposite one, expressions (2) are transformed in the following way:

$$\begin{array}{l} \mathbf{5f \rightarrow 6d \rightarrow 7p \rightarrow 8s} \\ \mathbf{4f \rightarrow 5d \rightarrow 6p \rightarrow 7s \rightarrow} \\ \mathbf{4d \rightarrow 5p \rightarrow 6s \rightarrow} \\ \mathbf{3d \rightarrow 4p \rightarrow 5s \rightarrow} \\ \mathbf{3p \rightarrow 4s \rightarrow} \\ \mathbf{2p \rightarrow 3s \rightarrow} \\ \mathbf{2s \rightarrow} \\ \mathbf{1s \rightarrow} \end{array} \qquad (3)$$

One can also rewrite relations (3) in the form

$$\begin{array}{l} \mathbf{5f \rightarrow 6d \rightarrow 7p \rightarrow 8s} \\ \mathbf{4f \rightarrow 5d \rightarrow 6p \rightarrow 7s \rightarrow} \\ \mathbf{4d \rightarrow 5p \rightarrow 6s \rightarrow} \\ \mathbf{3d \rightarrow 4p \rightarrow 5s \rightarrow} \\ \mathbf{3p \rightarrow 4s \rightarrow} \\ \mathbf{2p \rightarrow 3s \rightarrow} \\ \mathbf{2s \rightarrow} \\ \mathbf{1s \rightarrow} \end{array} \qquad (4)$$

Separation of odd and even lines gives

$$\begin{array}{ll} \mathbf{^{14}4f \rightarrow {}^{10}5d \rightarrow {}^{6}6p \rightarrow {}^{2}7s \rightarrow} & \mathbf{^{14}5f \rightarrow {}^{10}6d \rightarrow {}^{6}7p \rightarrow {}^{2}8s} \\ \mathbf{^{10}3d \rightarrow {}^{6}4p \rightarrow {}^{2}5s \rightarrow} & \mathbf{^{10}4d \rightarrow {}^{6}5p \rightarrow {}^{2}6s \rightarrow} \\ \mathbf{^{6}2p \rightarrow {}^{2}3s \rightarrow} & \mathbf{^{6}3p \rightarrow {}^{2}4s \rightarrow} \\ \mathbf{^{2}1s \rightarrow} & \mathbf{^{2}2s \rightarrow} \\ \textit{Cluster M} & \textit{Cluster F} \end{array} \qquad (5)$$

were the upper index indicates the number of elements constituting the corresponding subshell. Expressions (2-5) demonstrate the symmetry of even and odd lines, which have the identical structure and equal number of elements. Furthermore, a symmetry of subshells filling along the leading and secondary diagonals of expression (3) is evident. As a matter of fact, it is a 4-dimensioanl structure. Even and odd lines subdivide the set of all subshells into two classes. Four lines of each class (expression (5)) creates the second dimension. The third dimension is established by subshells which form a line and the fourth dimension is formed by the set of subshell elements: 2 elements of **s**-subshell, 6 elements of **p**-subshell, 10 elements of **d**-subshells and 14 elements of the subshells **f**. The sum of the principal **n** and orbital **l** quantum numbers of the terms in each line of expressions (2), (3) and (4) is constant, e.i. $\mathbf{n+l = const}$. For each column of the "cluster" M in the expression (5) $\mathbf{n - l = const}$. For each column of the cluster F in the expression (5) one also obtains: $\mathbf{n - l = const,}$ but the constant value is different. We see that the order of electronic subshells filling in coordinates $\mathbf{n + l}$ and $\mathbf{n - l}$ possesses a four dimensional symmetry. The graphical representation of expressions (2,3,4, 5) is given on **Fig.1.**

3. <u>The definitions of the elemental cell, the elemental family and the ensemble of elements</u>. One should pay attention to the fact that the shells, in formulas (2), (3), (4), (5), as well as on the Fig.1, are not present explicitly. **Fig.2** shows the shells which constitute the skeleton of the electronic energy levels filling in the hydrogen atom. The comparison Fig. 1 with Fig. 2 shows that **IN THE PROCESS OF FILLING OF THE ELECTRONIC LEVELS AS A FUNCTION OF THE NUCLEI CHARGE THE KEY ELEMENT IS A SUBSHELL RATHER THAN A SHELL**. Because the shells in chemistry play the accessory role only, it is reasonable to give to the subshell a name which sounds independent from the shell. The name ***electronic cell***, or (in brief) the ***cell***, sounds adequate. To the set of elements of a cell is given the name ***elemental family*.** The set of all chemical elements will be called as ***the ensemble of elements***.

<u>The definition of a level:</u> The lines of inequalities (2-5) were defined as **levels**. The number of elements on the levels are:

The 1st cycle:	2 elements, one cell (Hydrogen and Helium);
The 2nd cycle:	2 elements, one cell (Lithium and Beryllium);
The 3rd cycle:	8 elements, two cells: B, C, N, O, F, Ne (p-family) and Na, Mg (s-family);
The 4th cycle:	8 elements, two cells (p-family: Al, Si, P, S, Cl, Ar; and s-family: K and Ca);
The 5th cycle:	18 elements, 3 cells (d, p and s families);
The 6th cycle:	18 elements, 3 cells (d, p, s families);
The 7th cycle:	32 elements 4 cells (f, d, p and s families);
The 8th cycle:	32 elements, 4 cells (f, d, p and s families).

It is important to pay attention to the following features:

a) The distribution of the elements over the levels does not coincide with the classical elemental distribution over the periods that existed in the original Mendeleyev's periodic table;

b) The number of levels is one unit bigger than the number of periods in the periodic system;

c) The number of elements of every even level is equal to the number of elements of a preceding uneven level. Hence, the additional dimensional ***super cyclicity*** can be introduced into the periodicity of the elemental properties.

One can subdivide *the ensemble* of chemical elements known today into 19 families. However, the synthesis of 119 and 120 elements is the matter of time. When these two elements will be discovered, the 4d-pyramid will have eight closed levels and 20 cells.

<u>The definition of a cluster:</u> the elemenal ensemble can be distributed over 2 ***clusters***. The cluster that is formed by the elements of uneven levels will be defined as ***M-cluster***, the cluster that is formed by the elements of even levels will be defined as ***F-cluster***.

<u>The definition of a cycle:</u> We will call the levels which constitute each cluster as ***cycles***. Uneven levels (e.g. the levels of the M-cluster) will be called ***M-cycles***, while even levels (e.g. the levels of the F-cluster) will be defined as ***F-cycles.*** Each cluster consists of 4 cycles. The cycles that form a cluster will be called primary, binary, ternary and quaternary (according to the augmentation of the number $2(2\mathbf{k}+1)$ of elements in the cycle **k**).

<u>The definition of a supercycle:</u> The levels number $2\mathbf{k}-1$ and $2\mathbf{k}$, $\mathbf{k}=1,2,3,4$, which have equal number of cells (and equal number of family elements in each cell) constitute ***supercycle***. Each

supercycle consists of 2 cycles of equal length: one M-cycle and one F-cycle. The number of chemical elements which constitute the supercycle number **k** is $(\mathbf{2k})^2$.

4. FOUR DIMENSIONAL (FUNDAMENTAL) PIRAMIDS OF THE PROPERTIES OF THE ENSEMBLE OF ATOMS.

Four-dimensional pyramid of chemical elements in the level representation. All 118 chemical elements known today can be represented by 4-dimensional pyramid (**Fig.3a**). In this pyramid, cycles are disposed along the secondary diagonal (from up-right to left-down). All the cells constituting a level have been colored by the same color. The cell filling inside the level as a function of nuclei number occurs along the secondary diagonal of the pyramid in the direction from up-right to left-down. The cells (cubes) of the F-cluster are shifted deep down the picture by ½ of the cube edge. The elements of each cell, which do constitute the element's families, are disposed in the dimension orthogonal to the page plane.

Four- dimensional pyramid of chemical elements in the cluster representation: Fig. 3b.

Four -dimensional pyramid of chemical elements in the cupercyclic representation: **Fig. 3c**.

The subdivision of elements ensemble by a) levels, b) clusters or c) supercycles, indicated by colors, on Figs. 3a, 3b and 3c is different, as the shape of the 4-d pyramid in all three representations remains unchanged. The name ***fundamental pyramid of atoms and elements***, which designates the shape of pyramids (Figs. 3a, 3b, 3c) has been given, regardless to what classes the ensemble of elements is subdivided.

5. PHYS-CHEM (FUNDAMENTAL) ROTATION. Let's compare the electronic energy levels in the hydrogen atom (Fig. 2) with the cell filling as a function of atomic number (Fig. 1). One can note that the rotation the Fig. 1 by $\pi/4$, and subsequent reflection in the bisector of the first quadrant, given in Fig.4, makes the disposition of sets of the cells, depicted on these pictures, identical.

6. ELEMENTAL AND ATOMIC REFERENCE FRAMES. The representation of electronic cells (subshells) for the hydrogen atom is given in the coordinates **n** (principal quantum number) and **l** (orbital quantum number), see Fig.2. The rectangular part of the periodic tables of elements, by tradition which goes back to Mendeleev [9], is represented in the coordinates **n(l)** as well (**Fig.5**). This coordinate system will be identified as ***elemental coordinates.***

The representation of electronic cell filling as a function of atomic number has four-dimensional structure in coordinates **n+l** and **n-l**. That coordinates will be called ***atomic coordinates.*** This implies that:

A) the structure of electronic levels of the hydrogen atom., e.g. the one-electron problem (Fig.2),

and

B) the structure of filling of the cells of the ensemble of elements (Fig.1), e.g. many electrons problem,

can be transformed into each other by the transition from the elemental reference frame to the atomic reference frame.

7. PYRAMIDAL NUMBERS. Each element of the 4D-elements pyramid corresponds to the set of four pyramidal numbers, and this set is unique:

- *Cluster Number* μ, which values are M or F;
- *Supercycle Number* (or, in short, super-number) σ, which identifies the supercycle an element belongs to (σ = 1, 2, 3, 4).

- *Floor Number* ρ. The index ρ runs from 1 ("the ground floor" of the fundamental pyramid, which is constituted by alkaline and alkaline-earth metals) to σ (corresponding to the rear earth metals, which occupy the fourth, 'penthouse' floor of the fundamental pyramid).
- Family number θ, which enumerates the elements of the cell and adopts $2(2\rho - 1)$ values.

THE CERTAINTY PRINCIPLE: 4 pyramidal numbers determine each chemical element uniquely. The sets of pyramidal numbers for any two different chemical elements differ at least by one pyramidal number.

The certainty principle in this form is the analogue of the Pauli principle for the cyclically repeating properties of chemical elements.

8. UPPER FLOORS AND BOTTOM FLOORS OF THE 4D-PYRAMID. 4D pyramids of chemical elements have 4 floors. The elemental families which constitute the 1st, 2nd, 3rd and 4th pyramid ***floors,*** have s, p, d and f electrons on their outermost subshell correspondingly. One can notice that chemical elements, which maximum valency is growing from one (alkaline metals) to 7 (halogens), as well as inert gases (valency zero), occupy two ***bottom floors.*** All non-metals and semiconductors take up positions in the two bottom pyramid levels. Two other floors (***upper floors, or the penthouse***) of the fundamental elements pyramid are occupied by metals only.

9. ZIGZAG PATH. The order of filling of two bottom floors of the fundamental pyramid is presented in Fig. 5a. One can notice that the filling has a zigzag shape. The steps of the zigzag path marked red are carried out in the atomic reference frame, as the steps of the zigzag path marked violet are occurring in the elemental reference frame. The transitions of the zigzag path from level to level and from cluster to cluster occurs along the pyramid secondary diagonal (in the top-right – bottom-left direction). The pair of bottom-level cells disposed one above the other constitutes *the coupled cells*. Along the zigzag path motion, valency electrons are generated in the cell couples united by the red arrows (elemental reference frame), and never in the cells connected by the violet arrows (atomic reference frame). This property is a key one for the universe matter structure. If the zigzag path would have a different shape, or the cell coupling inside the zigzag path would had a different structure, the chemical compounds, e.g. the world we live in, would be different In particular, the carbon atom, which has been consisted of two s and two p orbitals, would not possess four bonds having almost identical shape and energy. The existence of the chains of atoms (in particular, nucleic acids and polypeptides) would be also impossible, as well as life itself.

10. PERIODIC TABLES AND THE ZIGZAG PATH. By tradition traced back to Mendeleev [9], all periodic tables are based on the elemental valence. The structure of the Periodic Table, based on the fundamental pyramid of atoms and elements, is depicted on Fig. 5b. Black arrows show the cells filling order in the periods 4-7. Black circles indicate the first and the last cell of each period. Red arrows indicate the order of the coupled cells of the zigzag path filling for the periods 4-7 (in the periods 2 and 3, the coupled cells of the zigzag path and the periods of the Periodic Table are identical. The elements of the first period (H and He) do not have coupled cells because they are constituted by one 1s cell only).

a) The first and the last cells of each period of the periodic table (black circles on Fig. 5b)
and
b) The coupled cells of the zigzag path (yellow arrows on Fig. 5b)
coincide

It was D.Mendeleev (9) who first suggested this selection of the starting point of the periods, which is generally accepted to our time. However, if one superimposes the periodic table on the 4d pyramid of atoms and elements, the traditional definition of periods looks tricky (Fig. 5b). Of course, the origin of any cyclically repeating process can be chosen at any point of the cycle. However, the selection of the starting point of periods for the entire periodic table (alkaline metals) provides the identity of the cells, originating and terminating the period with the coupled cells of the zigzag path.

The coupled cells (s and p subshells of two bottom floors of the elements pyramid) constitute the core structure of the periodic law based on the elemental valence. This is why the core of all periodic tables has the rectangular form, despite the fact that the number of shell elements as a function of the principal quantum number **n** is equal to $2\mathbf{n}^2$, e.g. is growing parabolically.

11. PHYS-CHEM CONTROVERSY. From the previous paragraph and Fig. 5b it is clear that, from physical standpoint, the selection of the starting point of the periods of the Periodic Tables is very unnatural. The cell filling in periods 4-7 makes the "chess knight's move" to the 4D-pyramid penthouse. In addition, periods 4-7 consist of the pieces of two 4D-pyramid levels, which belong to two different clusters. Finally, the traditional subdivision of elements by periods (which unites elements of the subshells **s** and **p** of the coupled cells in the period) destroys the four-dimensional representation of the elemental ensemble, in particular, its subdivision by levels, clusters, cycles and supercycles.
However, from the point of view of chemistry, the periodic table is perfectly logical. From chemical standpoint, the 4D-pyramid and its 4D-symmetries are much less important than the key property of chemical elements, i.e. the valence. The reason of this controversy lays in the fact that the elemental valence, as well as the periodic valency properties of the whole elemental ensemble, are based on the zigzag path. The elemental valence and the zigzag path are not revealed in the individual atoms at all, until the chemical bonds with other atoms are established. However, the imposition of the zigzag path on the 4D pyramid destroys the open-work 4-dimensional pyramidal structure (Fig. 5a). This is the core of phys-chem controversy, originating in the fundamental principles which determine the properties of elements (atoms).

12. THE FUNDUMENTAL ELEMENTS PYRAMID IN THE VALENCY REPRESENTATION. The rotation of the elements (parallelepipeds, *'pyramid bricks'*) of two bottom floors of the pyramid from the direction orthogonal to the plane to the page plane, generates ***'Valence Elemental Building'*** (**Fig. 6**). The transition along the zigzag path on the 4D pyramid (Fig. 5a) corresponds the movement on Fig. 6 vertically from its bottom to the top starting with the hydrogen atom (shown in the right corner of the first floor of the pyramid). After the upper floor of the Elemental Valency Building is reached, the zigzag path goes to the element of the first floor on the left side from the vertical line which has been passed, etc. Note that all non-metals, semiconductors and rare gases are disposed on the bottom floors of the 4D pyramid. Thus the rules of 4d pyramid of atoms and elements filling on one hand, and the rules of the Periodic table filling as a function of the elements number on the other hand, coincide.

The valency in the eight-floor element building is increasing monotonically from the bottom to the top, were the shells have been closed by the rare gases. The «8-floor elemental building» represents the key principle of the classification of the periodic table. The elements

groups (from 1 to 8) are disposed vertically, as the periods (from I to VII) have been depicted horizontally. This representation is based on the elemental valences only, hence the rectangular form of the periodic systems of elements [9] becomes explicit.

13. FUNDAMENTAL PYRAMIDS IN THE ATOMIC REFERENCE FRAME. The fundamental 4D-pyramids (Figs. 3a, 3b and 3c) have been raised from the elemental reference frame i.e. in the coordinates **n** and **l**. The fundamental pyramid in the atomic reference frame, i.e. in the coordinates **n+l** and **n-l**, is erected in **Fig.7a**. Here, the levels are disposed horizontally, as the cells capable to form the combined chemical bonds are disposed along the secondary diagonal. The zigzag path has been also transformed accordingly.

The zigzag path on the 4D-elements pyramid, erected in the elemental coordinates system, is depicted on Fig.5a**.** The same zigzag path shown on the 4D-pyramid erected in the atomic coordinates system is depicted on Fig.7a.

14. 3.5-DIMENSIONAL RERPESENTATIONS OF THE PYRAMIDS OF ATOMS AND ELEMENTS: THE «*ELEMENTAL STADIUM*» AND THE «*ELEMENTAL SKYSCRAPER*». Four-dimensional pyramids of the elements adequately represent the periodicity of the properties of atoms and elements. The access to any element of the pyramid on the computer screen can be easily programmed. However, on paper the visualization of a number of elements, disposed inside the pyramid volume, is impossible. The representations, which demonstrate the 4D periodic properties of chemical elements and make all elements visible, would be very useful. Such representations were named **3.5 dimensional** [5]. The swinging of the elements families of the fundamental pyramid (Fig. 7a) from the dimension orthogonal to the page plane into the direction parallel to that plane generates the 3.5-D pyramid in the atomic representation, or '***The Stadium of Atoms***' (**Fig. 7a)**. Similar swing of the elemental families of the fundamental pyramid (Figs.3) and disposing them vertically in the page plane, generates the 3.5-D pyramid in the elemental representation, or **"The Skyscraper of Elements" (Fig. 8b).**

15. PYRAMID CLOSINGS. There are three different ways of the elemental pyramid "closing".

1) The shell closing occurs vertically, from the ground floor (i.e. first from the bottom) to the pyramid top (Fig. 1). The top floor number k is closing the shell number k (Figs. 3). The closing of shells in the ensemble of all elements is depicted in Fig. 9a. The plane (black) indicates that the corresponding shell closure is possessing the symmetry. Note that the symmetry of the closed odd levels (e.g belonging to the M-cluster) is a mirror one, as the symmetry of even pyramid levels closings is a combinational symmetry of the mirror reflection in the plane AND the transformation of M-cluster into F-cluster. Therefore, the shift of the corresponding cube by ½ of its edge back from the picture plane must be substituted by the same shift forward from the picture plane, and vice versa.
2) The level closing. The filling of orbitals occurs along the secondary diagonal, from the pyramid top to its bottom (Figs.1 and 3). A level closing made by the alkaline-earth metals, having two s-electrons on the outermost cell. The terms of atoms whose levels are closed have been depicted in Fig. 9b.
3) The period closing (with an inert gas) by 6 electrons of the p-cell. The filling starts on the 1st pyramid floor and terminates on the 2nd pyramid floor, in periods 4-7 making chess-knight move along the way (Figs. 5b). The period closings are depicted in Fig. 9c.

In addition, one can also consider the supercycle closure as a distinct one from the closure of levels. It is similar to the level closing but takes place in the even levels only. The elements with closed supercycles are depicted in Fig. 9d.

In Fig. 10, all four closings are superimposed on the fundamental pyramid. The cells which close periods, shells and levels are designated by red, green and blue circles correspondingly. The cells, which close supercycles, are designated by large circles. The direction of filling is indicated by arrows: the filling of the groups of the periodic table is designated by red arrows, the filling of shells is indicated by green arrows, the filling of levels (supercycles) is indicated by sky blue and dark blue arrows correspondingly. Note that:

- The closing of shells, levels and periods occurs in different cells.
- The only cell, in which all three closings have been united, is the cell **1s**.
- On the 4d-pyramid surface, the angle between the directions of a level filling and a period filling is equal to ¾ π.

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FIGURE LEGENDS

Fig.1 The filling of the electronic cells (subshells) as a function of the element number.

Fig.2 The filling of electronic subshells of the hydrogen atom.

Fig.3a The fundamental pyramid of elements in the level representation.

Fig.3b The fundamental pyramid of elements in the cluster representation.

Fig.3c 4D-pyramid of elements in the supercyclic representation.

Fig.4 **Phys-Chem (fundamental) rotation.**

(a) electronic levels for the hydrogen atom.

(b) the rotation by $\pi/4$ which formally transforms the set of electron energy levels in the hydrogen atom into diagram of the cells filling depicted in Fig.1, which reveals 4D-structure.

(c) the reflection of Fig 4b in the bisector of the first quadrant, which reveals the similarity and difference between the set of hydrogen atom electron levels and the filling of the subshells of elements as a function of the atomic number.

(d) The split of all subshells (cells) by clusters (blue and pink circles) and supercycles (designated by red, yellow, blue and violet lines).

Fig. 5a **The zigzag path** which determines the elements valence. The 4D-pyramid is depicted in the level representation. The yellow segments designate the order of the cells filling within the period. The brown segments indicate the order of the filling of the cells, which belong to the different periods. The cells of the bottom floors of each level are colored with the identical color. The upper floors of all levels are colored gray in order to indicate that all elements of the pyramid "penthouse" are metals, which do not reveal any periodically repeating valency.

Fig. 5b **The structure of the Periodic Tables drawn on the 4D-Pyramid of Elements.** Black arrows show the cells filling order in the periods 4-7 of the periodic tables. Black circles indicate the first and the last cells of each period. Red arrows indicate the order of the coupled cells of the zigzag path filling for the periods 4-7 (in periods 2 and 3 the filling of coupled cells of the zigzag path and periods coincide with each other). Note that

- the first and the last cells of the period 2-7 of the Periodic Table

and

- the coupled cells of the zigzag path

coincide.

Fig. 6 **The Elements "Building and the Roof".** The element families of two bottom floors are turned into the page plane. In this representation, maximum valence of elements is increasing from 2 to 7 in the floors from 2 to 7. The 8^{th} floor is occupied by the inert gases. This representation of the elemental pyramid reveals the groups and the periods of the two-dimensional periodic systems originated by Mendeleev, but destroys four-dimensional structure of the properties of the elements ensemble.

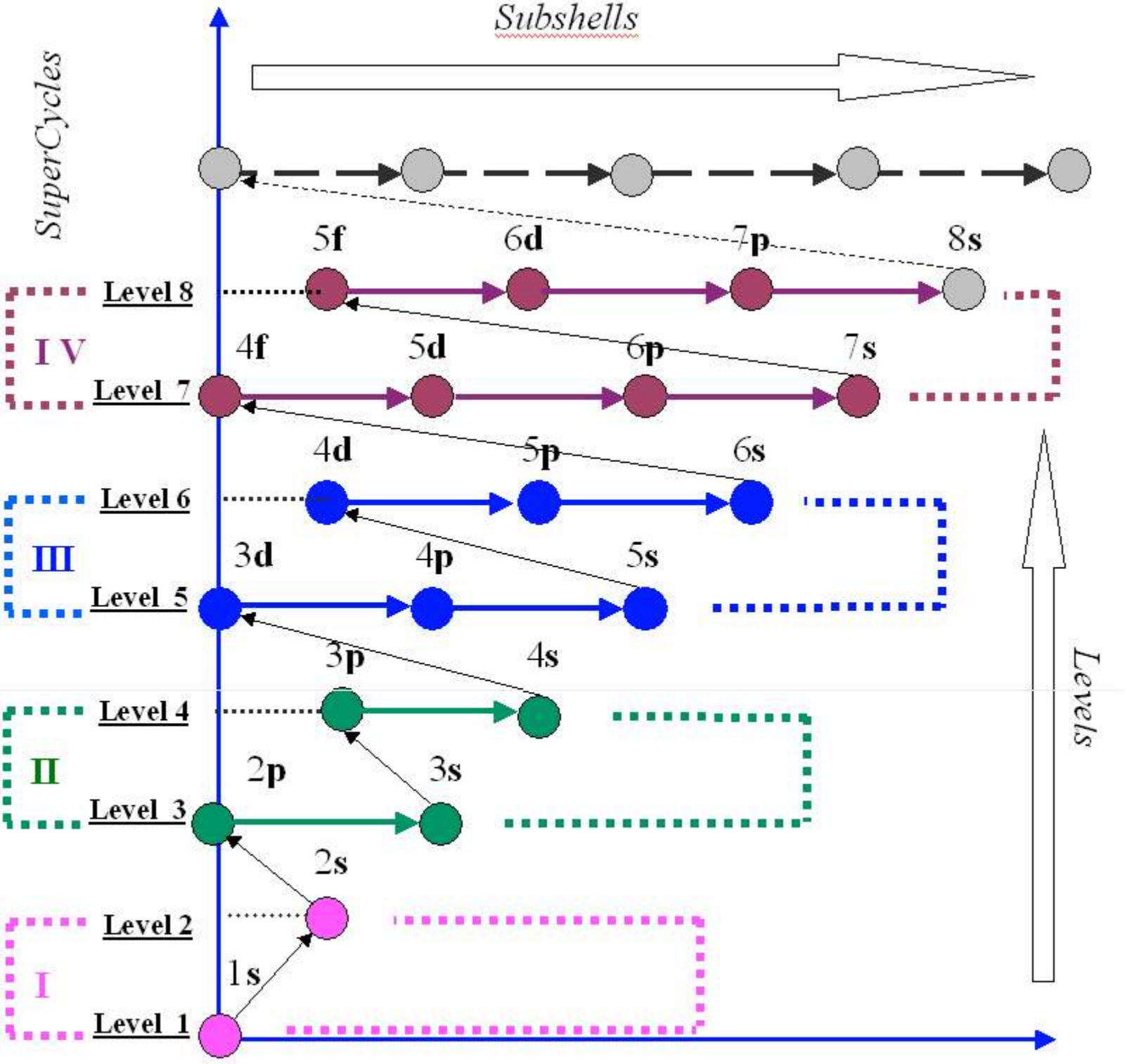
Fig. 7 The 4-dimensional pyramid in the atomic representation. Here the levels are disposed horizontally rather than along the pyramid's secondary diagonal (Fig.3). Inside each level s, p, d, f cells occupy positions in the order, from left to right. Two upper (penthouse) floors of the pyramids 3 (built in the elemental representation) now (in atomic representation) are on the right side of each level (gray). The colors of the segments of the zigzag path are identical to these on Fig. 5a.

Fig. 8 **(a)** 3.5-dimensional pyramid of elements in the atomic representation ("atomic stadium").

Fig. 8 **(b)** 3.5-dimensional pyramid in the elemental representation ("the skyscraper of elements").

Fig. 9
(a) **The shell closing.**
(b) **The level closing.** The plane indicates the symmetry of the closed terms of M-cluster elements (on the right) and closed terms of F-cluster elements (on the left) correspondingly.
(c) The periods closing.
(d) The closing of the supercycles.

Fig. 10 Four closings (shell, level, periods and supercycles) depicted on the face surface of the fundamental pyramid of the elements. The cells, which close periods, shells and levels are designated by red, green and blue circles correspondingly. The cells, which close supercycles, are designated by large circles. The direction of filling is indicated by arrows: the filling of the groups of the periodic table is designated by red arrows, the filling of shells is indicated by green arrows, the filling of levels (supercycles) is indicated by sky blue and dark blue arrows correspondingly.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SuperCycles
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6d
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Level 6
III
3d
4p
5s
Level_5
3p
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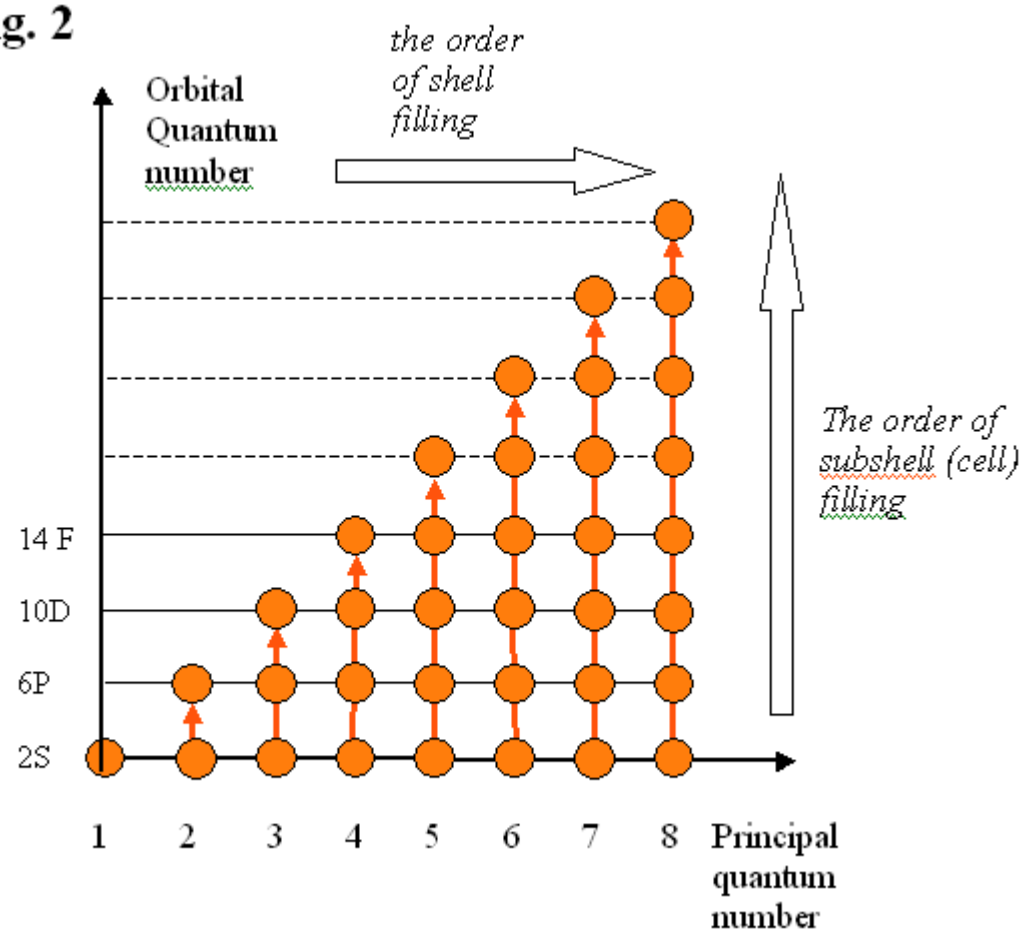
Levels

Fig. 1

Fig. 2


the order of shell filling
Orbital Quantum number
The order of subshell (cell) filling
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10D
6P
2S
1
2
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4
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6
7
8
Principal quantum number

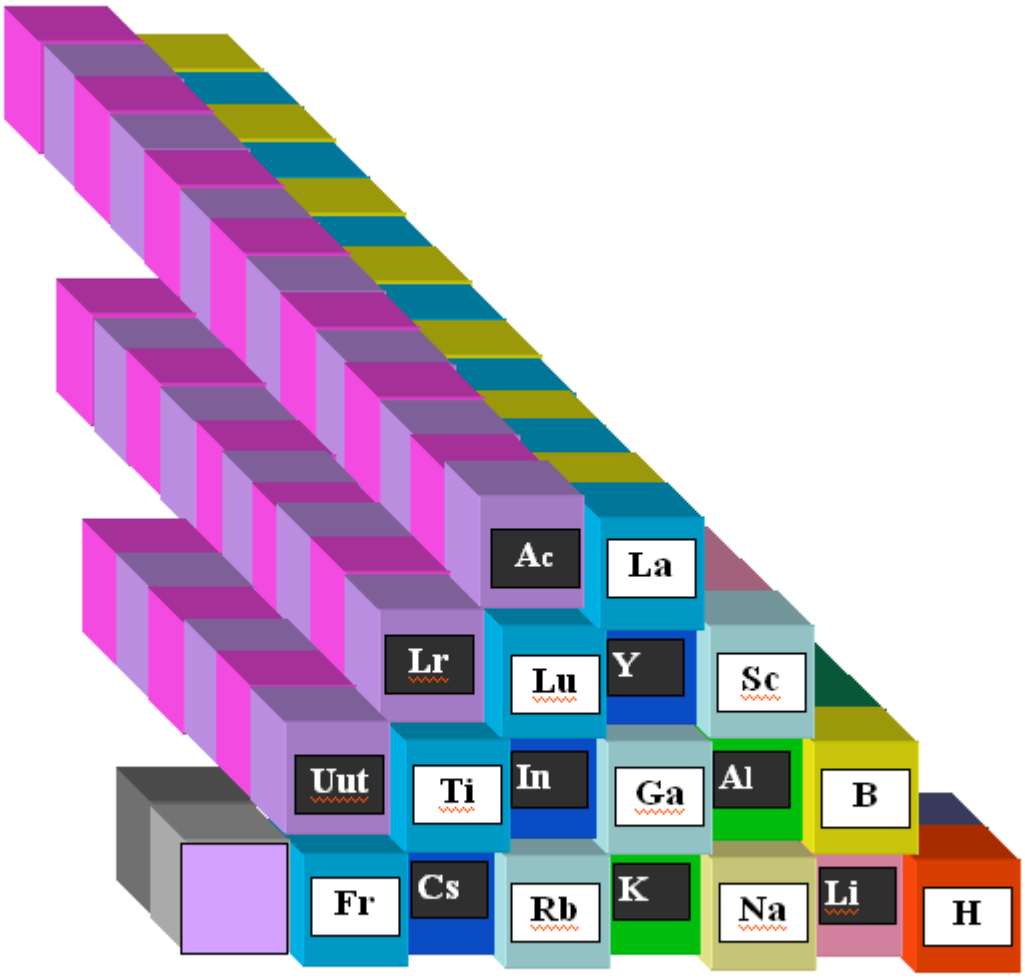


Fig. 3a

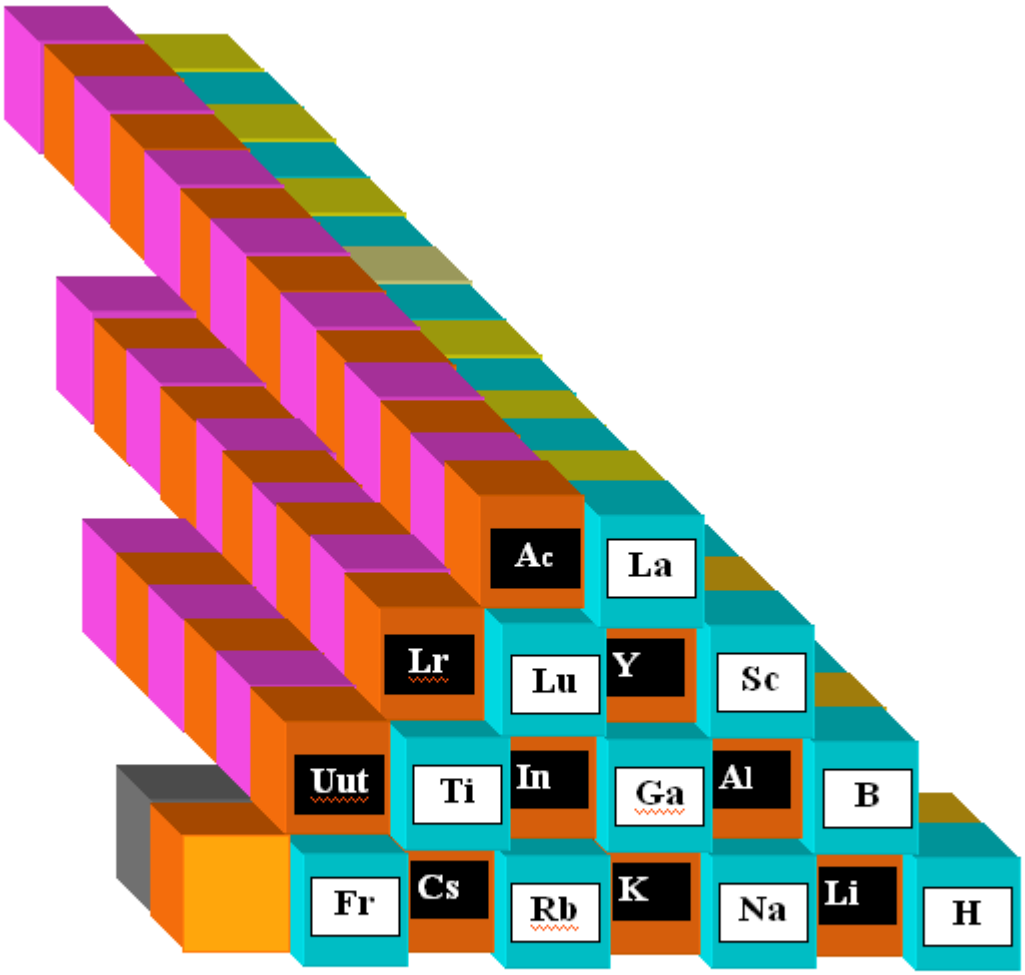


Fig. 3b

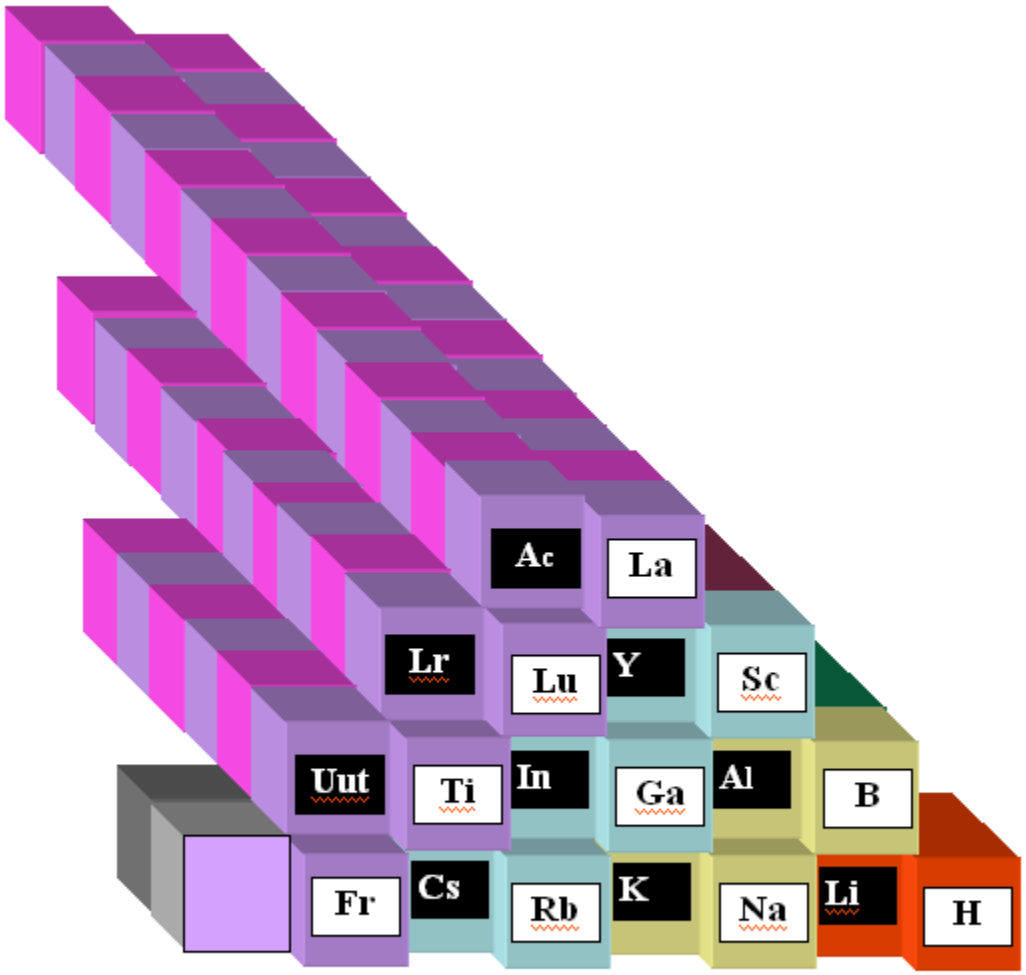


Fig. 3c

Phys-Chem Rotation

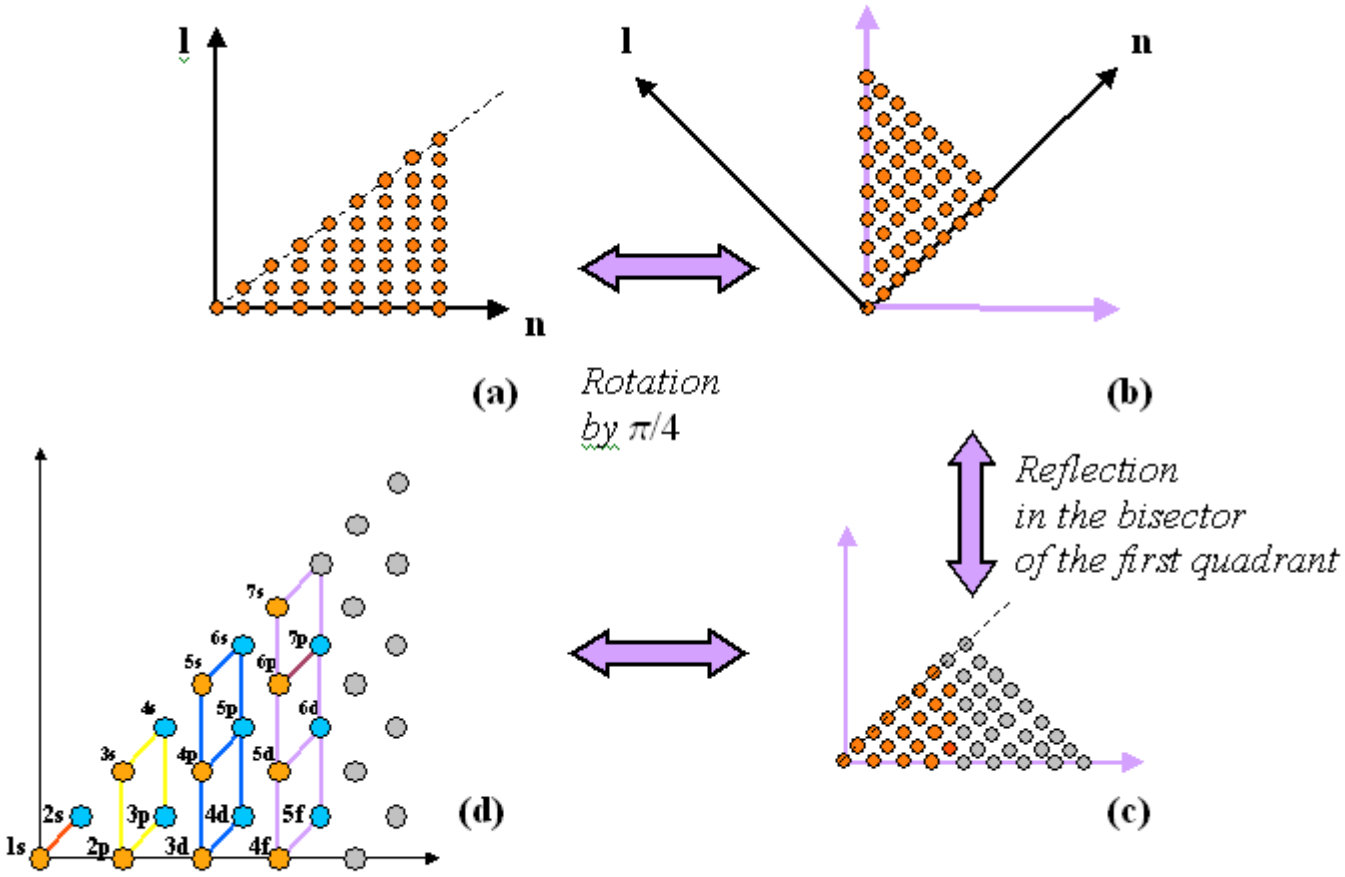


Fig.4

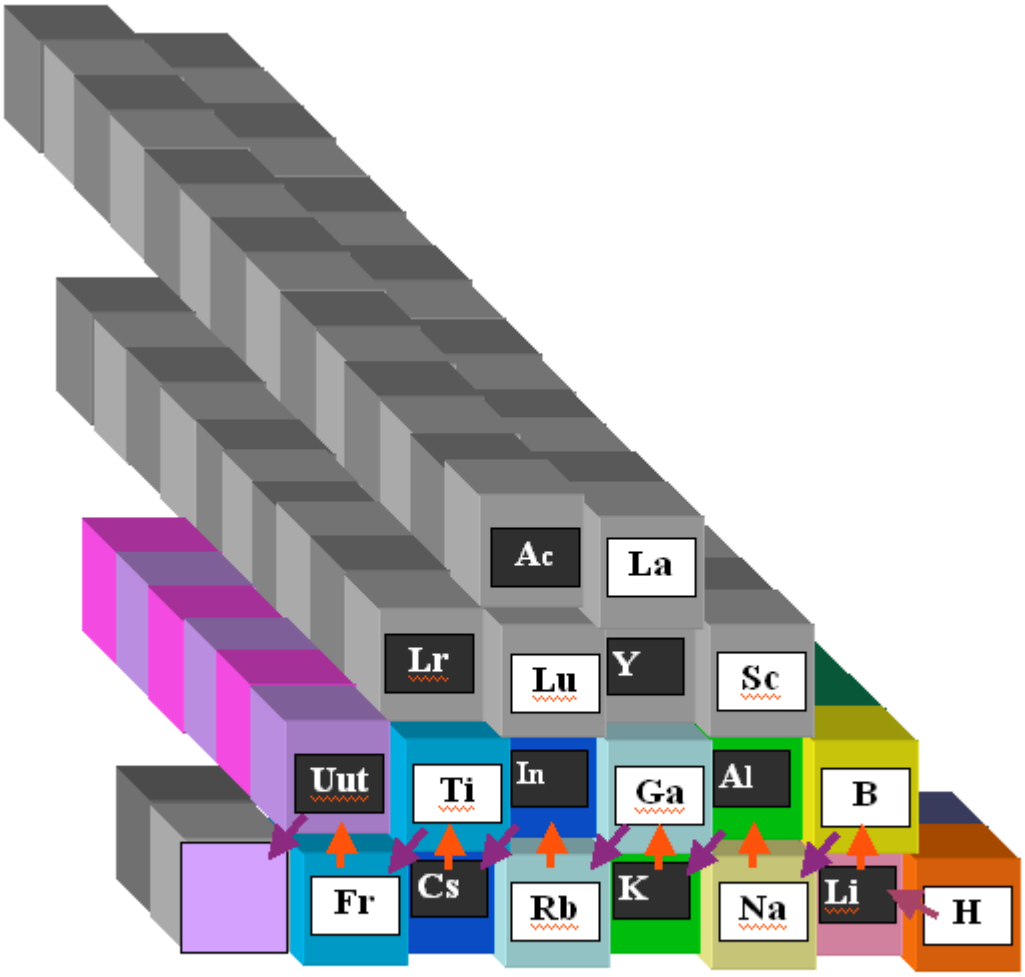


Fig. 5a Zigzag Path

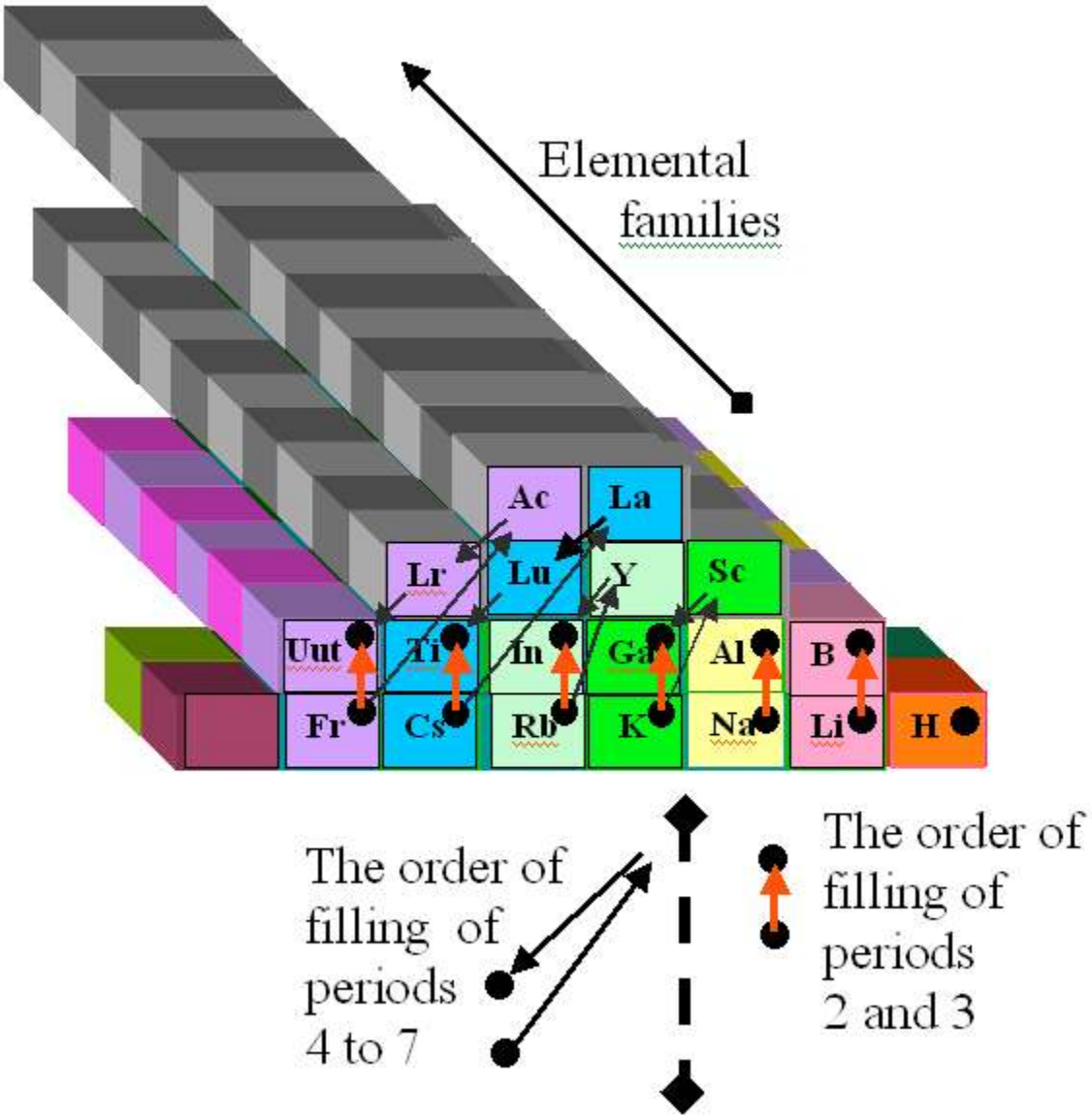


Fig. 5b

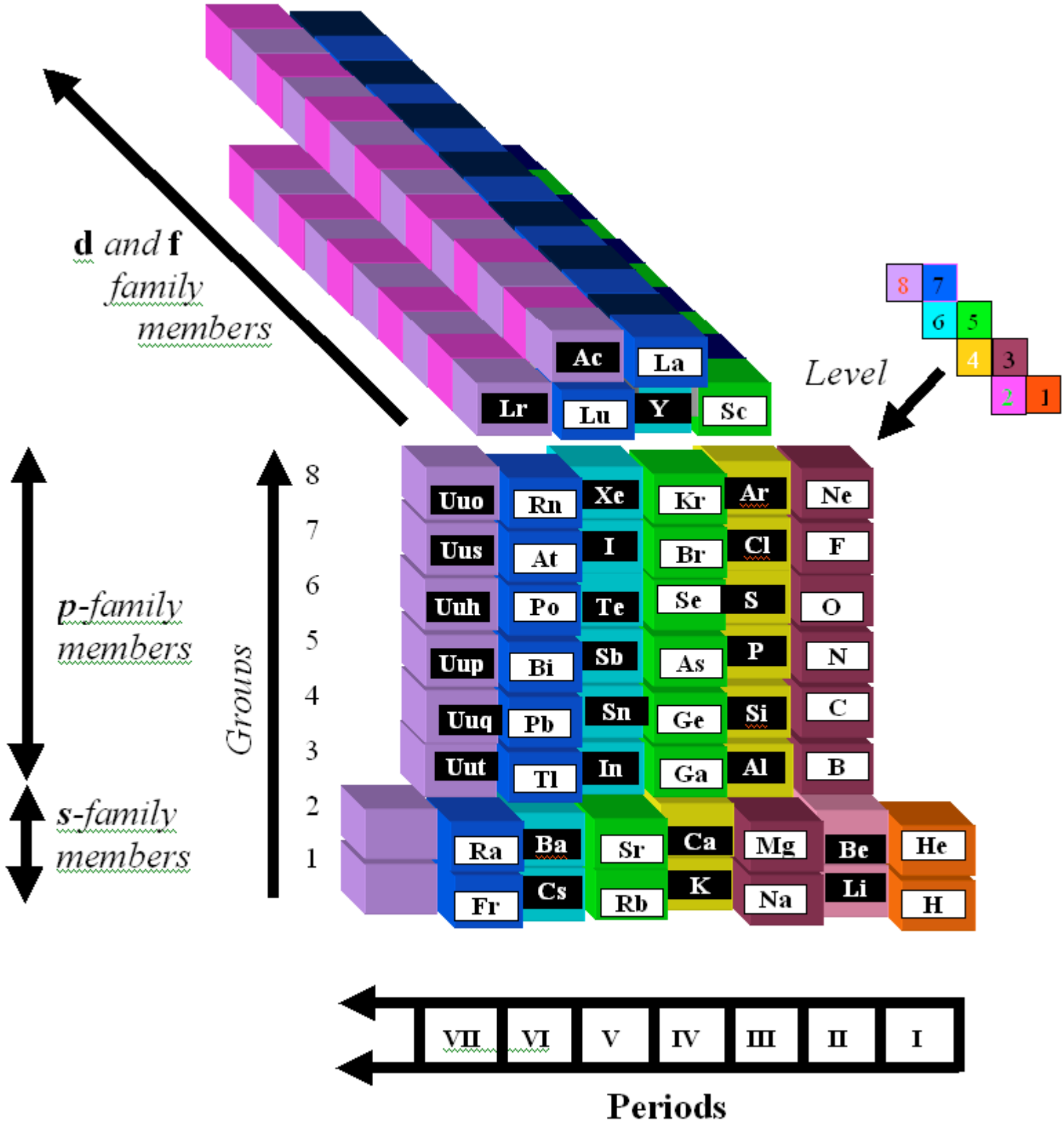

d and f family members
Level
8
7
6
5
4
3
2
1
Ac
La
Lr
Lu
Y
Sc
p-family members
s-family members
Groups
8
7
6
5
4
3
2
1
Uuo
Rn
Xe
Kr
Ar
Ne
Uus
At
I
Br
Cl
F
Uuh
Po
Te
Se
S
O
Uup
Bi
Sb
As
P
N
Uuq
Pb
Sn
Ge
Si
C
Uut
Tl
In
Ga
Al
B
Ra
Ba
Sr
Ca
Mg
Be
He
Fr
Cs
Rb
K
Na
Li
H
VII
VI
V
IV
III
II
I
Periods

Fig. 6

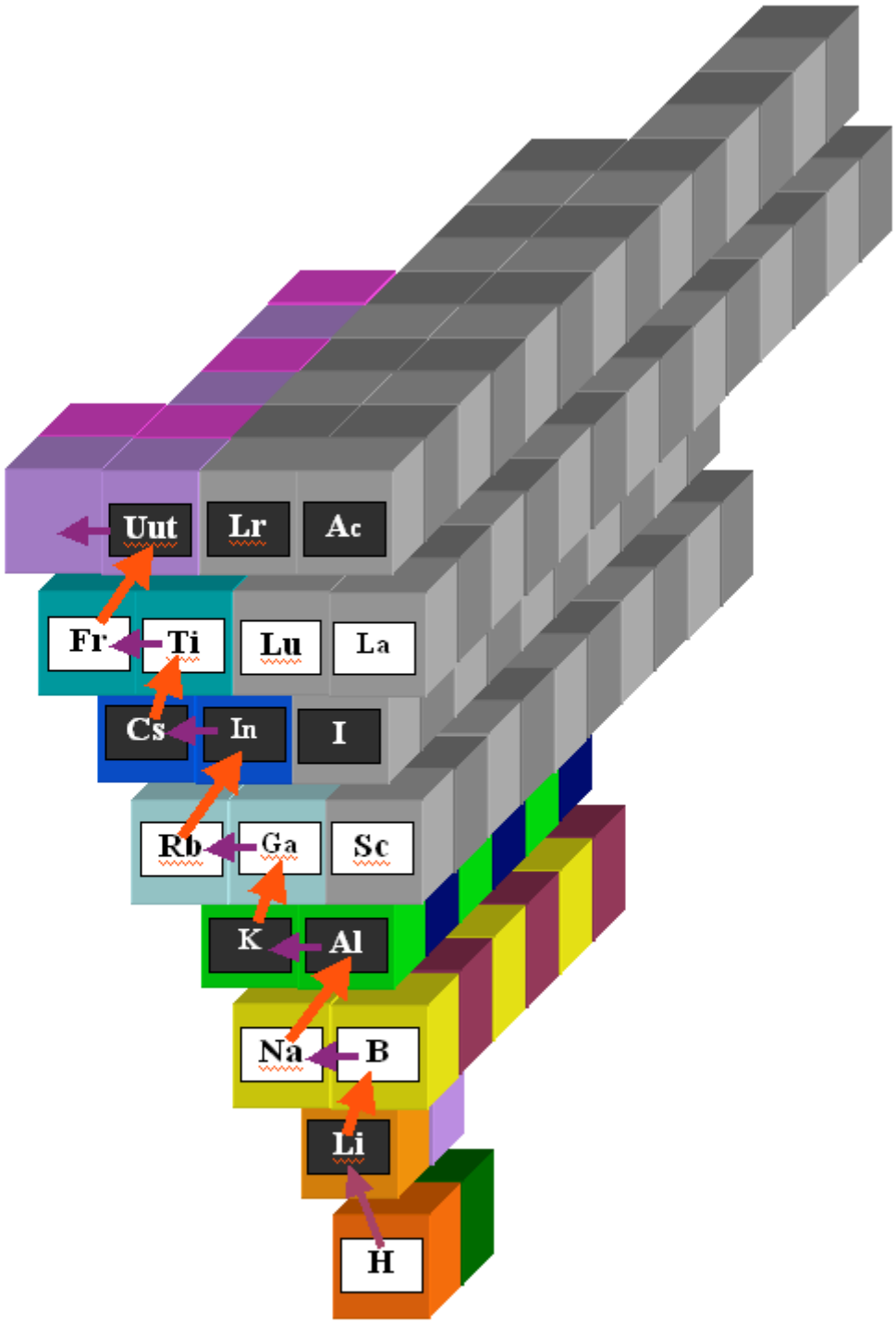
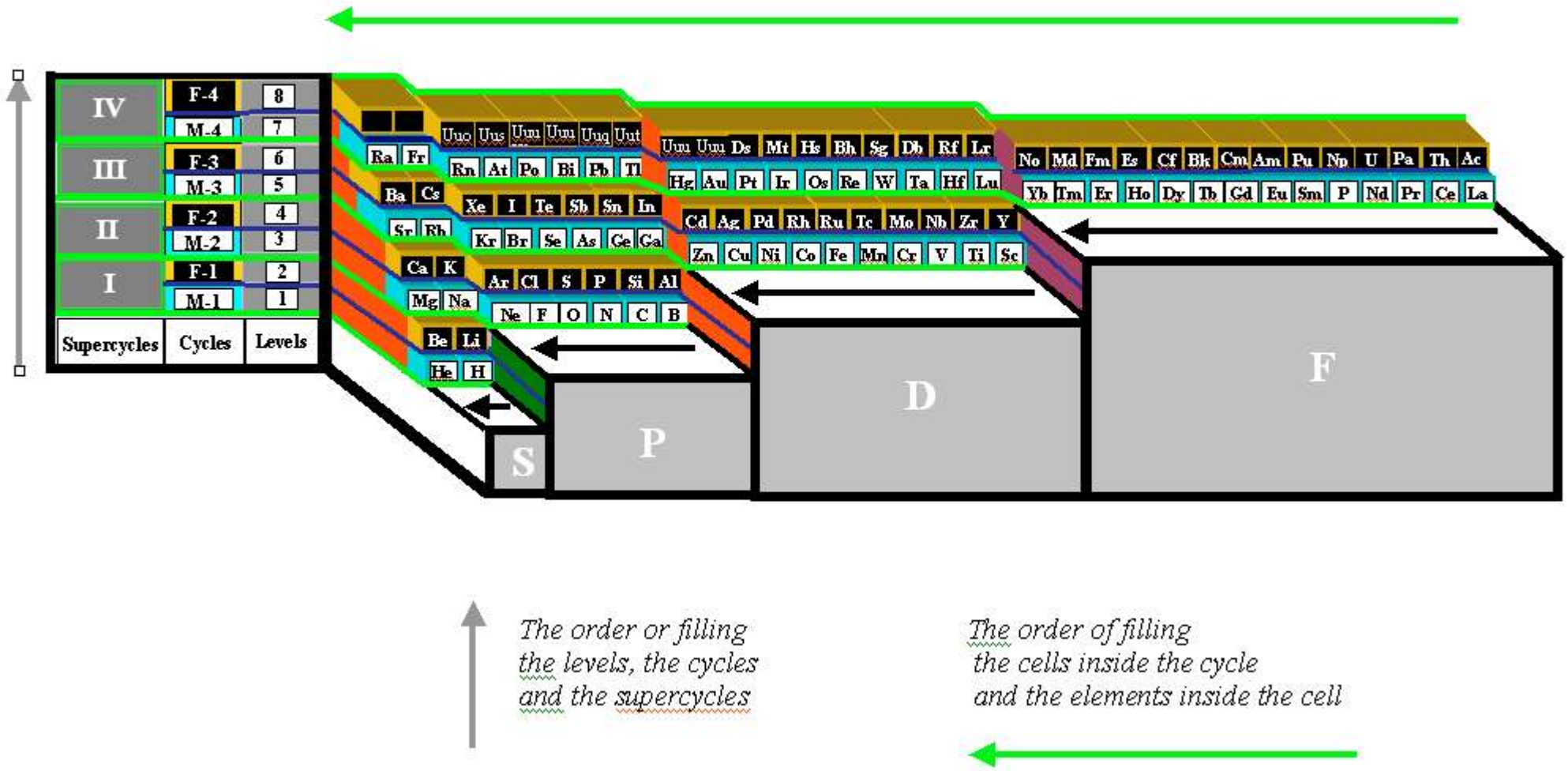


Fig. 7

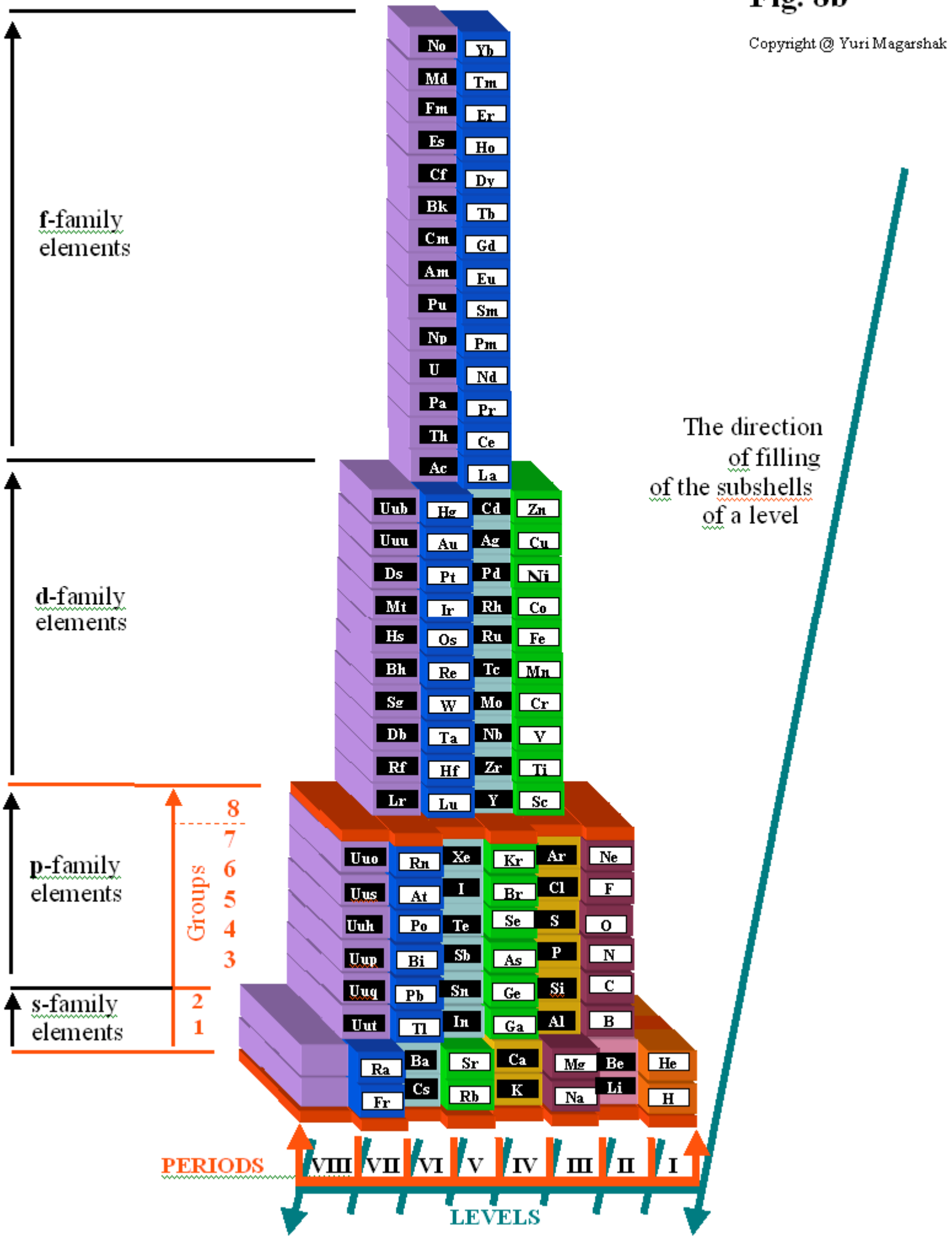
Fig. 8a: "Atomic Stadium"



The order or filling the levels, the cycles and the supercycles

The order of filling the cells inside the cycle and the elements inside the cell

Fig. 8b



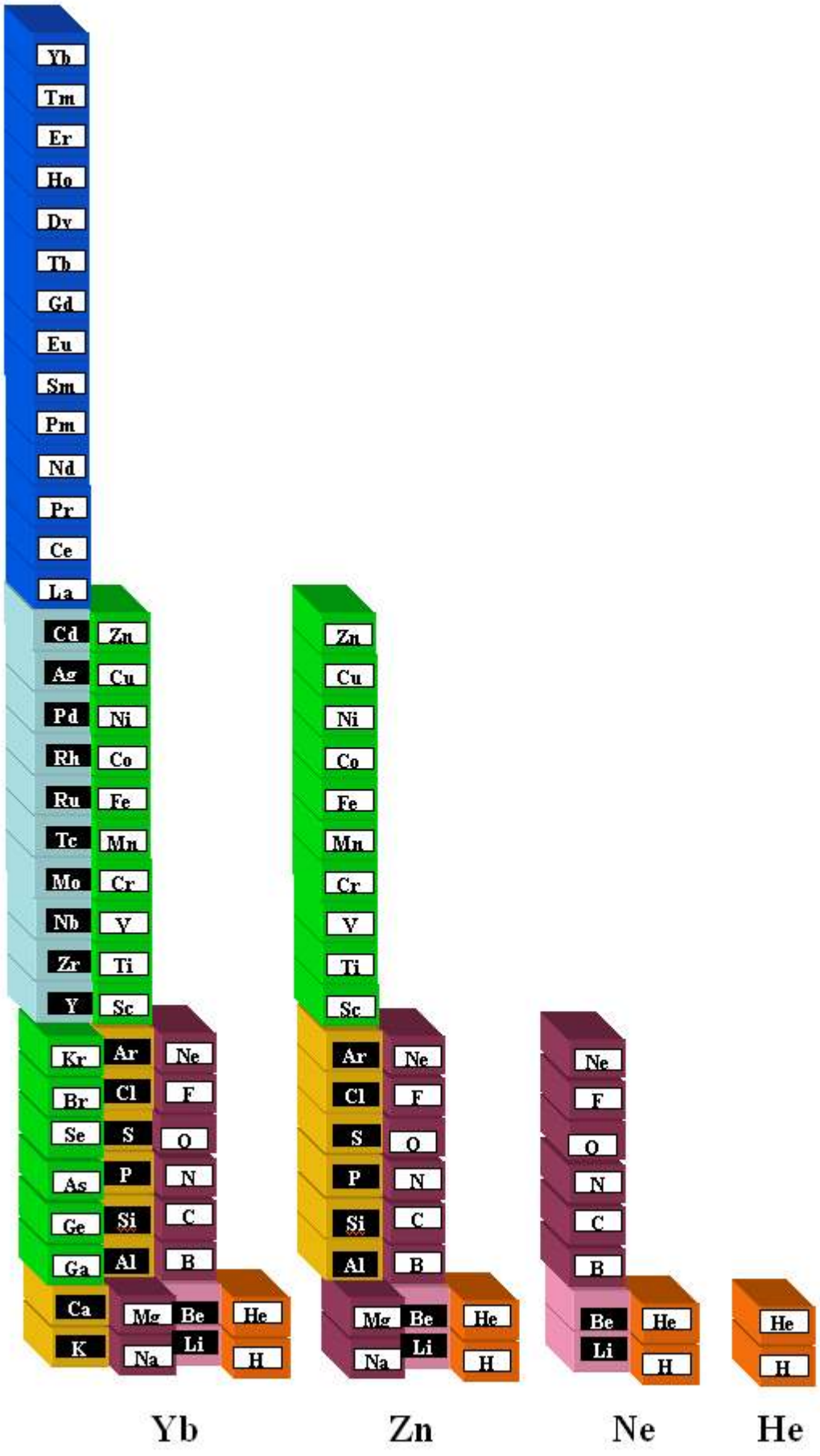


Fig. 9a: Closed Shells

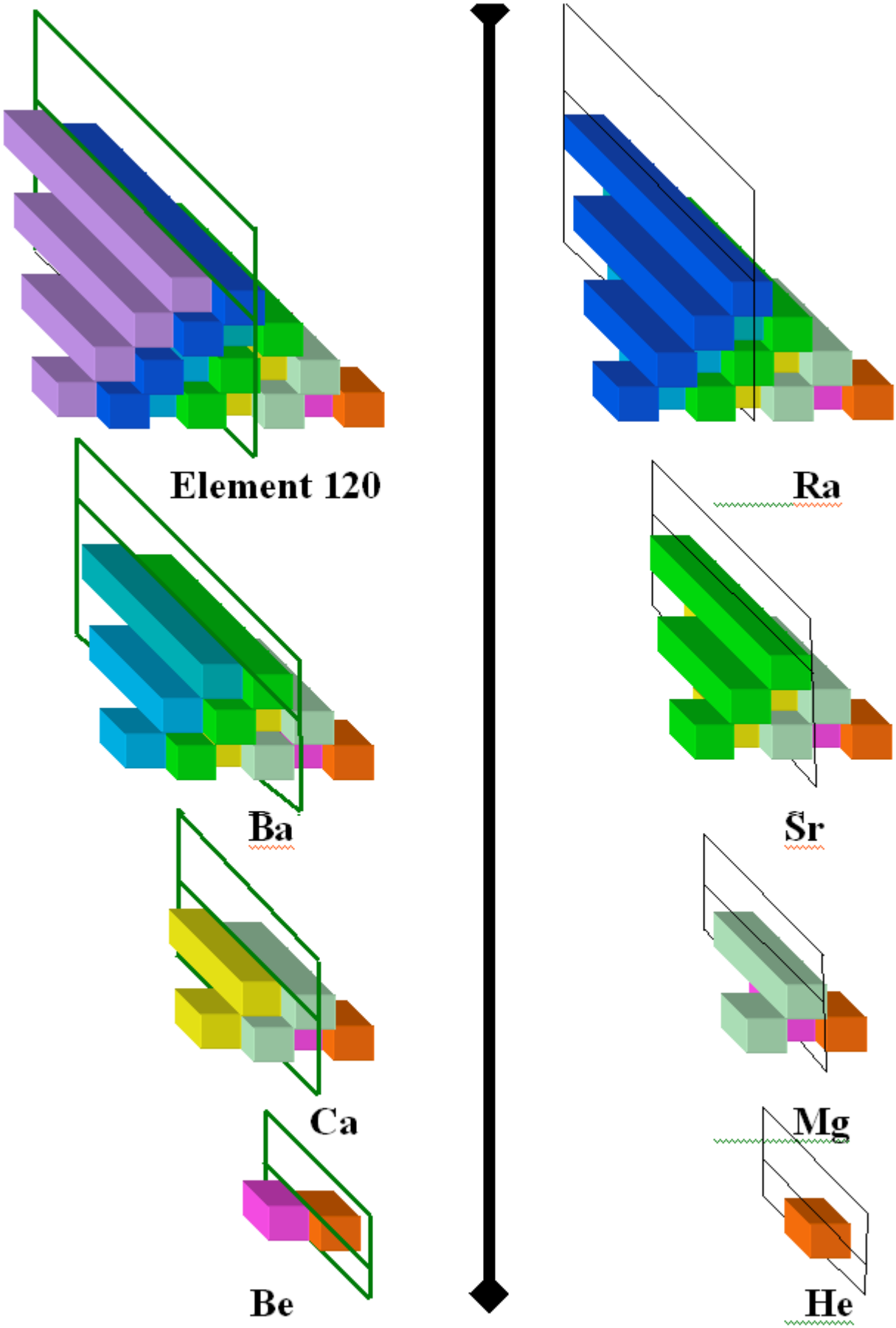
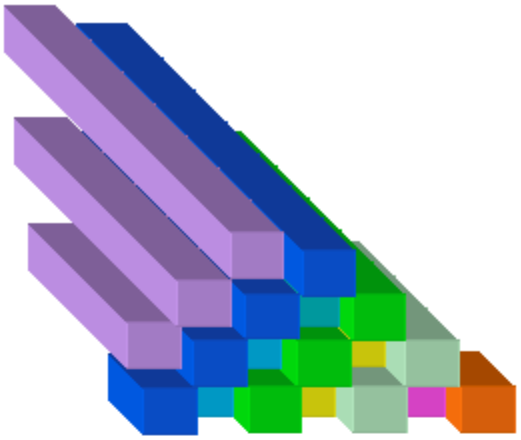
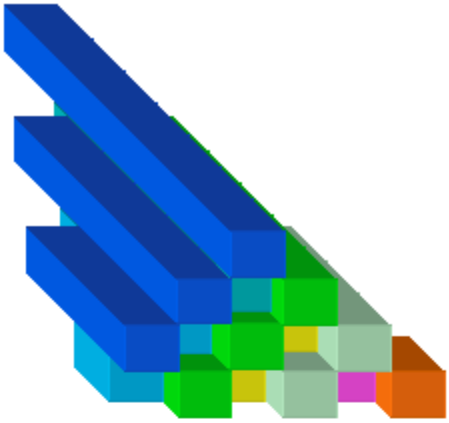


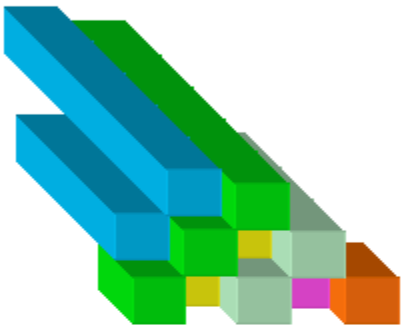
Fig. 9b: Closed Levels



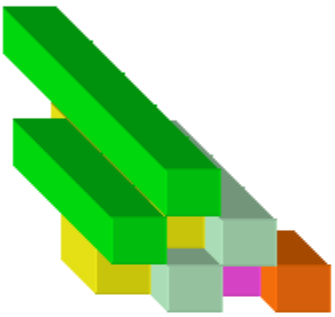
Uuo



Rn

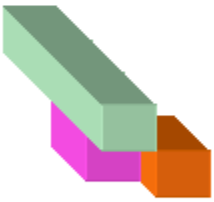


Xe



Kr

Ar



Ne

He

Fig.9c: Closed periods

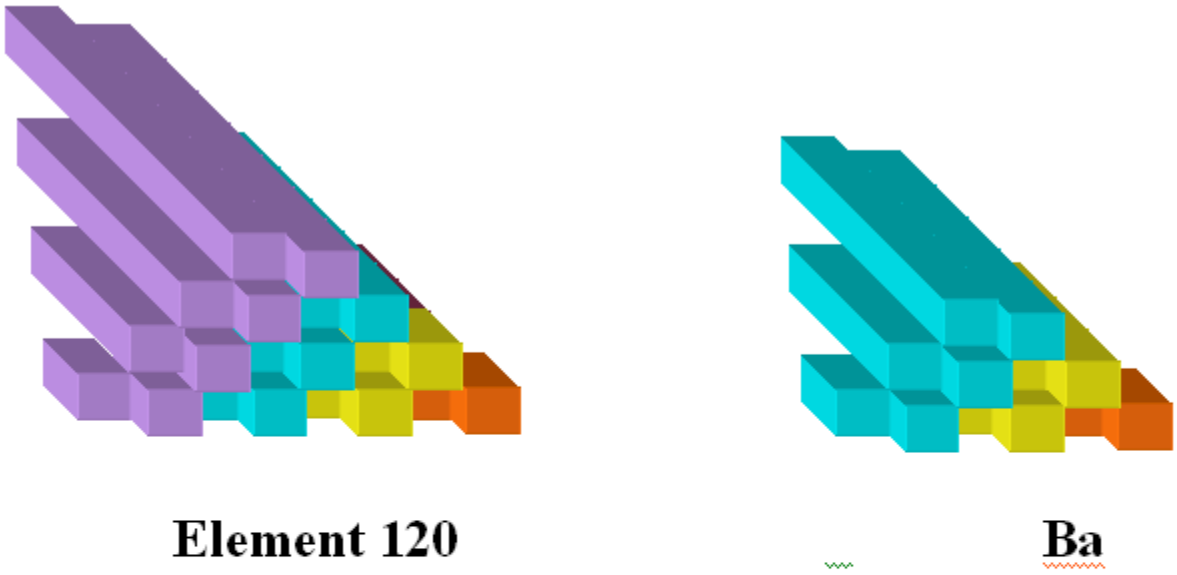


Fig.9d: Closed Supercycles

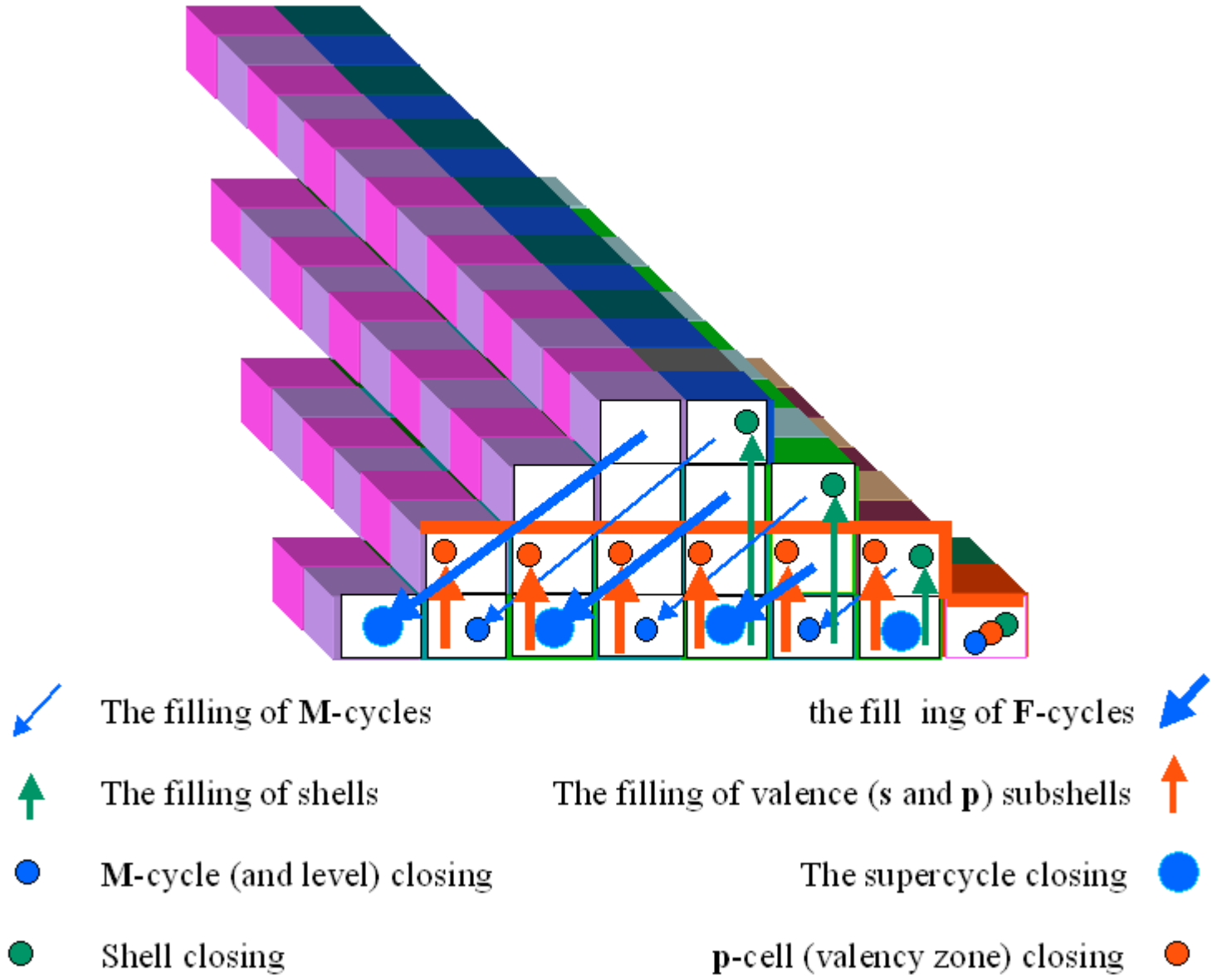

The filling of M-cycles
the fill ing of F-cycles
The filling of shells
The filling of valence (s and p) subshells
M-cycle (and level) closing
The supercycle closing
Shell closing
p-cell (valency zone) closing

Fig.10