Mathematical proof of the possibility of representation of functions of complex variables by colors of color circle

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May 1997-July 2023

ABSTRACT. The creation of theories of human visual perception has more than a century of history [Ref. 1-8]. As shown in 1996 [Ref 9], there is a one-to-one correspondence between the field of complex numbers and colors perceived by humans. Examples of color representations of functions of a complex variable are shown in Figures 1-4. This article provides a rigorous mathematical proof of this phenomenon.

MATHEMATICAL SUBSTANTIATION OF FORMALISM.

Let G be a finite group and R[G] be the ring of a regular representation of G over real numbers. Let $R^{+1}[G]$ be a positive octant in R[G]. It consists of linear combinations \$\Sigma⊲ a_i g_i, a_i $R_{, a\_i}$ 0, g_I G. It is clear that $R^{+1}[G]$ is invariant under summation and multiplication. R[G] contains R as a subring of constant functions on G. In fact, R[G] decomposes into a direct sum of the matrix rings of three types : M(n,R), M(l,C), M(s,H) where r, C, H are the rings of complex numbers and quaternions respectively. and M(n,R) is the ring of n times n - matrices over R and similarly the other ones are the rings of matrices over C,H-respectively.

Consider the projection R[G] to R[G]/R where R is a G-invariant subspace generated by ⊲ g_i, g_i G Then the map $R^+[G]$ to R[G]/R is surjective. Indeed for any x= ⊲ a_i g_i we can find m > 0 such that a_i +m >0 for any i. This gives a representation of any element x in R[G]/R as the image of some element x+me $R\wedge^+[G]$.

Corollary. Any algebra in the above decomposition of R[G] can be realized by positive elements with linear equivalence. In particular if $G=Z_3$ then R[G]/R = C and we obtain realization of complex numbers by triples of positive real numbers. Similarly if G $=Q_8$ is a quaternion group generated by i, j, k, ij = k = -ji, jk =i= -kj, ki =j =-ik ,i^2=j^2=k^2= -1. In this case have a natural decomposition R[Q_8] = H + R +⊲ R_i were we make a summation over 3 nontrivial Z_2-characters of Q_8. The elements of Q_8 form a natural system of 8 vectors in H so that any element can be presented uniquely as a sum of four linearly independent elements from Q_8 with nonnegtaive coefficients. Similar picture holds for the representations R[G/H]. The group acts by permutations on the set G/H and R[G/H] always contains G-invariant subspace R. The map of $R^{\{+\}}[G/H]$ \$ to R[G/H]/\$ is always surjective.

Example Any system of basic roots of semisimple Lie algebra provides with a similar property for the corresponding real Cartan sublagebra k. If e_i are such roots then any element in k has a unique representation as a sum of e_i with nonnegative coefficients. Note that the first example related to Z-3 corresponds to the root system A_2.

It sounds natural to give these matrices a special name. Because the matrices on cones in general correspond to idea of complementarity (nucleotides, colours, etc.) we give them the name **complementary matrices**. For instance, in case of 3D-cone-representaion of algebra of complex numbers one has one complementary projection, but in case of 4D-cone-representation of algebra of complex numbers one has two complementary projections, one for real and one for imaginary numbers. One can note that formalism is using well known fact that the sum of all matrices of a regular representation of any finite group is the matrix with all elements equal to one (and its complementary projection is zero) [Ref 10].

References

1. W. Peddie **Colour vision**, Edward Arnold & Co, London (1922).
2. G.McCollum, A.Pellionisz, Parsons J.H., **An Introduction to the study of Colour vision,** Cambridge (1924).
3. Land E.H., **Experiments in colour vision**, Sci.American 200, 2-14 (1959)
4. T.N., Cornsweet **Visual Perception**, Academic Press (1970).
5. Padgham C.A., Saunders J.E., **The Perception of Light and Colour**. Academic Press, New York (1975).
6. R.Llinas, **Tensorial approach to colour vision**, J.Theor.Neurobiol. 2, 23028 (1983)
7. Faber Birren **Colors perception in Arts**, Schiffer book (2021)
8. Swan M. Burnett **Theories of color-perception** (2022)
9. Y.Magarshak, **Projective geometry of color vision, generating the field of complex numbers**, Biophysics, 41, 3, 734-743 (1996).
10. Cartan E. **Theory of Spinors**, Dover, NY (1990).

Figures

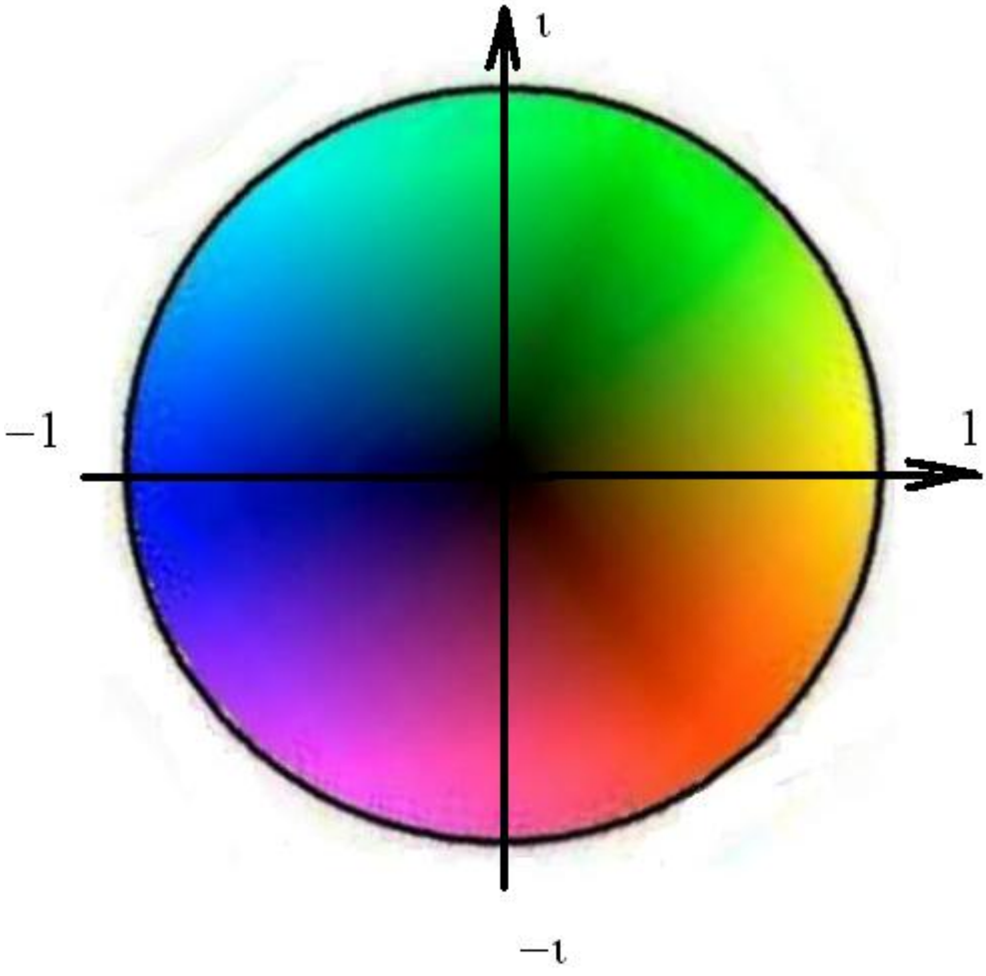

Fig 1 Color circle in the representation of complex numbers by colors

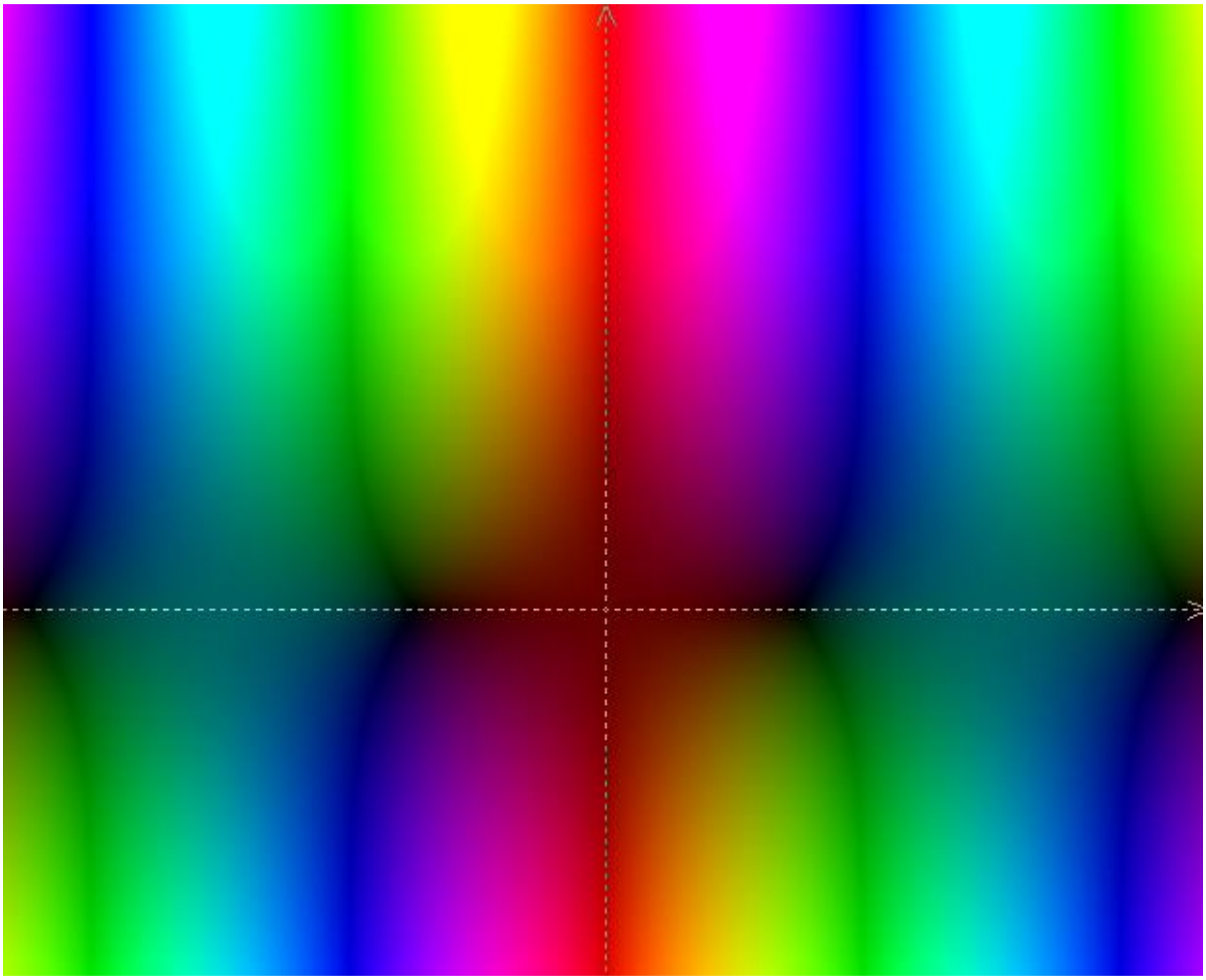

Fig 2:   Color representation of a function of a complex variable cos(z)

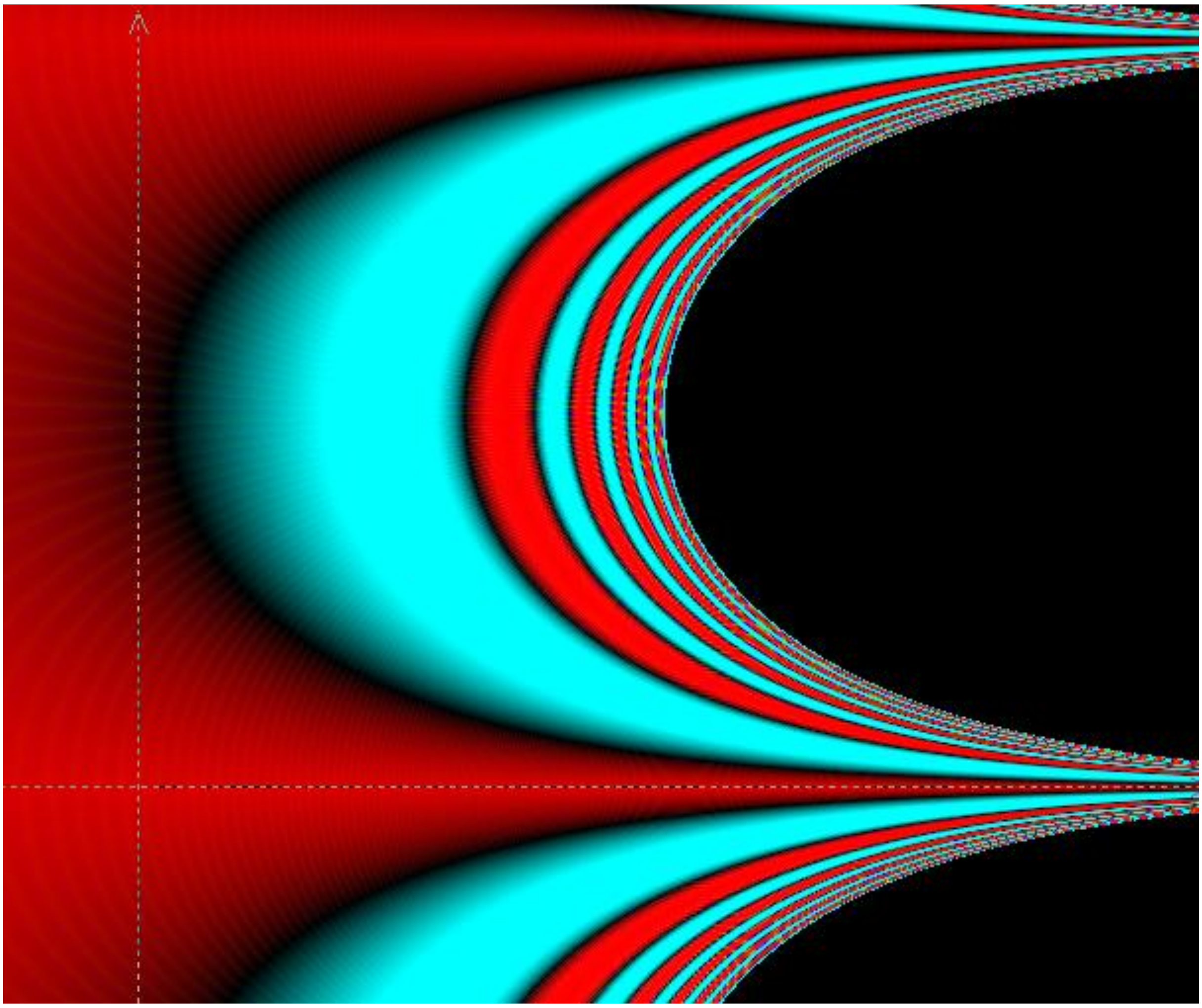

Figure 3. Color representation of a function of a complex variable
Cos (Sin (Log 10 (z) + 4 e $^{(z+3)/200}$))

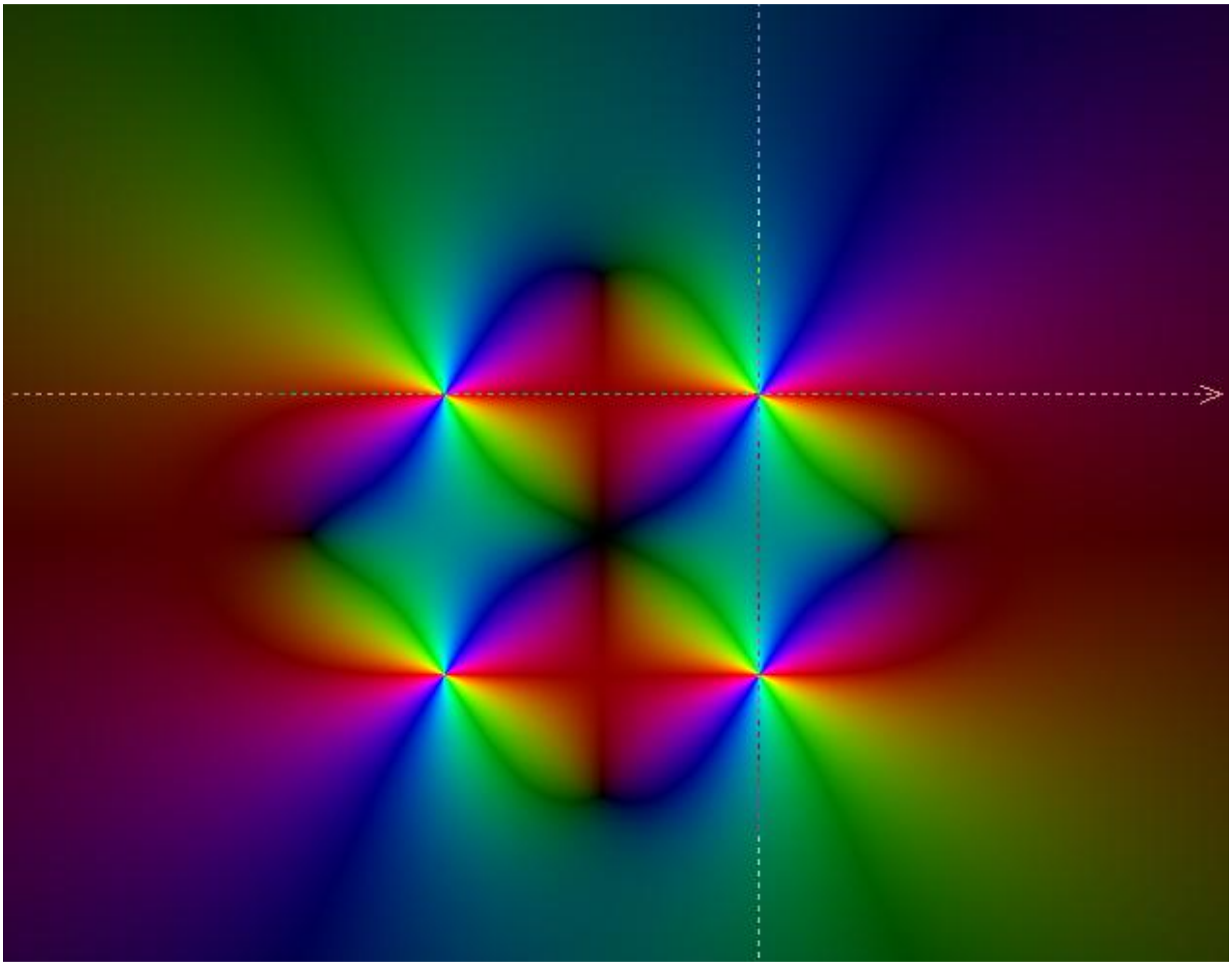

Fig.4. Color representation of a function of a complex variable
$1/z^2 + 1/(z+1)^2 + 1/(z+i)^2 + 1/(z+1+i)^2$